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THE MATHEMATICS TEACHER

Official Journal of the National Council of Teachers of Mathematics

Devoted to the interests of mathematics in Elementary and Secondary Schools

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THE MATHEMATICS TEACHER

Volume XLI Number 8

Edited by William David Reeve

Review of Guidance Report—Commission on Post-War Plans*

By Morris Krugman

Assistant Superintendent in Charge of Guidance

Board of Education, New York City

The guidance report of The National Council of Teachers of Mathematics is a unique document in several ways. In the first place, it is the product of a professional organization whose members are both mathematicians and educators. This gives the report professional status in both areas. In the second place, it reverses the usual process by considering the vocations in which mathematics is employed, mathematics being the frame of reference. In most occupational studies, the occupation would be the focus, and the treatment of the subject matter incidental. The report is unique in other ways. It is written for high school students. Other reports have, of course, been written for high school students, but they have usually been watered-down, over-simplified affairs, with very little content and a great deal of inspiration. This report is heavily weighted with content; inspiration is not lacking, but is subtly interspersed.

One fault of the report may be its all-inclusiveness. In trying to be all things to all men, it becomes difficult to treat every aspect equally well. The ten parts of the report really comprise three major divisions: mathematics for general use; mathematics for non-professional vocational use; and mathematics for professional use. The committee was undoubtedly aware of the problem of writing for three distinct types of readers and probably decided that for its purposes the disadvantages were outweighed by the advantage of a comprehensive treatment. In trying to write down to the non-mathematical mind, the result is sometimes objectionable to the others. The result is therefore uneven writing that may detract from the report as a whole. It is conceivable that superior high school students would be antagonized by over-simplified writing and the purpose of the report minimized for them.

Against this, however, is the fact that in about 25 pages almost every type of high school student can find material applicable to him. For further information he is referred to his mathematics teacher, to the guidance counselor, and to the bibliography. Since this report was obviously intended as a guidance publication and not as a treatise on mathematics, this approach is justified.

* Paper read at the Joint Luncheon and Panels of the Association of Chairmen of Departments of Mathematics and the Association of Teachers of Mathematics of New York City at Teachers College on March 13, 1948.

The writing in the report is irregular and gives evidence of participation by many hands. Though generally well-written, there are some spots that are not. English instructors will wince at occasional ungrammatical construction, and if the report is to have another printing, the simple corrections should be made. Some purists may object to the numerous "cute" expressions.

I know from personal experience that it is very difficult to write a report like this, and very easy to pick small flaws in it. In mentioning some faults, I have no intention of tearing down an outstandingly superior report. I am merely trying to present a balanced picture. In this vein, let me mention one or two errors in guidance procedures. There are several references to correspondence courses. Studies indicate that most people who enroll in correspondence courses do not complete the courses and do not get adequate returns for money spent. Guidance workers are therefore wary of recommending such courses. Then, once or twice, there is misleading information: one instance is in relation to entrance requirements for medicine, and another in connection with women in mathematics. In the latter, the statement is made that "at one women's college every mathematics major had her choice of 25 jobs in industry and government." This statement is not challenged as to veracity, but it referred to a war situation, and also had partial reference to nonprofessional work for which college work probably was not necessary- clerical work and some types of computing work, among others.

From the point of view of mental hygiene, I would like to see two items toned down. One is the constant emphasis on "fine personal qualities." Fine personal qualities are, of course, important, but the constant repetition would tend to discourage adolescent boys and girls who may have these qualities, but actually have feelings of inadequacy. This is not uncommon among intelligent, sensitive youngsters. The other is the treatment of ability. Ability is used synonymously with mathematical sense or, perhaps, with successful achievement in mathematics. "Low ability" is used in connection with the poor student of mathematics, and high ability for the reverse. This is, of course, a very incomplete concept of ability. From a mental health point of view, it would be desirable not to make students feel completely inferior if they have not been successful in mathematics. We know that some students with high ability fail for various reasons—sometimes for personal and emotional reasons, and sometimes because of some weakness in preparation or in teaching. Although this need not be emphasized, neither should lack of success in mathematics be attributed to poor ability on the part of the student.

A few words about the use of the report in guidance. This report makes an outstanding contribution to the vast amount of occupational information material available to guidance personnel for use with students. So much material exists, that the average guidance worker cannot possibly keep up with all of it. Without positive efforts, it will probably take years before it percolates into general use. It isn't enough to place it on sale and expect requests for it to pour in. Key persons throughout the country, both in mathematics and in guidance need to be made aware of its existence. Libraries—particularly high school libraries—should have copies on their guidance shelves. Announcements should be run in publications usually read by elementary, junior high, and senior high school teachers and by guidance workers—in such journals as *The Elementary School Journal*, *Clearing House*, Occupations, and the Science Research Associates publications.

One of the places where occupational information is most effective is in the group guidance room, which is almost universal in junior high schools, though not quite so common in senior high schools. Teachers and counselors in group guidance classes

are constantly on the lookout for pertinent occupational information material. Furthermore, eighth grade elementary school pupils, and 7th, 8th, and 9th grade junior high school pupils are the potential senior high school population, so that it is even more important to reach them than the senior high school students.

One effective method of acquainting large professional groups with a publication is that of sending review copies to key periodicals. This serves not only to arouse discussion about the publication, but also to have the title placed on bibliographies in the field. Bibliographies are cumulative in their effect and therefore give widespread coverage. These methods of communicating information are, of course, known to all. I mention them only as reminders.

Some educators question the advisability of presenting occupational information to students emphasizing one field only. Their objection is based on the supposition that a one-sided picture is presented to youngsters incapable of discrimination. The objection is valid if the material presented is over-glamorized and misleading, and if it is presented without supplying a picture sufficiently balanced to enable the students to arrive at a logical educational and vocational choice. This report does not have the faults mentioned—it is neither glamorized, over-enthusiastic, nor overly partial to mathematics. Mathematics is not over-sold; its place in our present day culture is frankly and objectively stated. Facts are presented, and the reader is invited to draw his own conclusions. This is in keeping with the accepted practice in guidance—occupational information is presented so that the student may be enabled to make a choice, but the ultimate choice is left to him and his family.

May I close by repeating something I said earlier. The guidance report of the Post-War Commission is an important addition to the literature of educational and vocational guidance. The few unimportant weaknesses mentioned detract but little from the report as a whole and from its usefulness. It is an extremely difficult type of report to write because it must survey a vast field; must be brief yet thorough; must be written at the level of the younger of the high school students; and must adhere to sound guidance principles. I believe the commission has done an admirable job in all of these.

Notice!

Miss Agnes Herbert, chairman of the arrangements committee for the Annual Meeting of the National Council of Teachers of Mathematics at Baltimore which begins on March 30, 1949 has appointed a committee on "Instructional and Learning Aids." It is the purpose of this Committee to display at the meeting devices which the mathematics teachers throughout the country have found helpful. If any of the readers have such devices, please send them to the chairman, ALFRED E. CULLEY, Forest Park High School (406), Chatham Road and Eldorado Ave., Baltimore, Maryland, not later than March 1, 1949. Materials can be claimed at the Lord Baltimore Hotel after the meeting on Saturday, April 2nd, or on Sunday, April 3rd. Materials not claimed then can be secured through the chairman at the Forest Park High School address.

Measures for Predicting Success in a First Course in College Mathematics

By M. W. Keller and H. F. S. Jonah
Purdue University, Lafayette, Ind.

Introduction

The sectioning or grouping of students taking required college mathematics courses is now a relatively common procedure. O'Quinn[1] reports that twenty-seven state universities, out of forty-three replying to his questionaire, used some method for grouping students according to mathematical ability or previous training in mathematics. The criteria used for grouping or sectioning students in mathematics courses were almost as varied as the number of universities which replied that they sectioned their students. In some universities all the students take the same course regardless of the group to which they were assigned, while in others those with the poorer preparation and/or ability were placed in special courses. In general, those universities which favored grouping were also strongly in favor of making adjustments in the amount of material. and type of subject matter to be taught in the different sections. From the O'Quinn report it is evident that there is a rather definite trend in the state universities toward the grouping of students according to ability and/or training.

The placing of students taking college mathematics into proper groups which apparently is becoming more prevalent. Hence this raises the question as to what criteria should be used to best perform this divisioning. One fairly satisfactory procedure is to obtain a regression equation based on various measurable factors which influence the student's grade and use this to determine his probable success in a given course. (There is, however, a considerable amount of disagreement on what measurable factors should be considered in the development of the necessary regression equation.) The student is then assigned to a course or section on the basis of this prediction. Although such a procedure is satisfactory it is rather involved. Consequently, it is more convenient to use a single criterion for this purpose if one can be found that is relatively reliable.

Several studies have been made to determine the effectiveness of various factors for predicting a student's success in mathematics. In a study at the University of Oregon, C. F. Kossack[2] states that of the different factors he considered for determining a student's probable success in a first course in college mathematics, the two most important factors were the student's grade on a placement or training test, and his high school mathematics score. He found that the score on a psychological test, the scholastic high school rank, and the number of years since graduation were not important.

Scott and Gill[3] report that of the two factors, number of units of high school mathematics and number of years intervening between the last year of high school algebra and entrance into college, only the number of units of high school mathematics was significant in predicting probable success in college mathematics.

Because of the general belief among college teachers of mathematics that the number of years since graduation, or what

[1] O'Quinn, Ralph, "Status and Trends of Ability Grouping in the State Universities." The Mathematics Teacher, Vol. 33, pp. 213–215, 1940.

[2] Kossack, C. F., "Mathematics Placement at the University of Oregon." *The American Mathematical Monthly*, Vol. 49, pp. 234–237, 1942.

[3] Scott, W. M. and Gill, J. P., "A Prediction of Pupil Success in College Algebra." The Mathematics Teacher, Vol. 34, pp. 357–359, 1941.

is roughly equivalent, the number of years intervening between the last course in high school algebra and entrance into college, is an important factor in predicting a student's probable success in a first course in college mathematics, the results of these two studies are significant and rather surprising. Each tends to corroborate the other. Other studies with different groups are needed, however, to verify these results.

From an extensive study on 891 engineering students, Irick[4] concluded that the score on the mathematics placement or training test was the best single factor for predicting a student's success in a first course in college mathematics. High school grades and rank in the high school graduating classes were next in predictive value. The number of semesters of mathematics taken in high school was next. The scores on an English placement test and on a psychological test were of relatively little value.

Dichter[5] states that the scores for 2466 students on the mathematics section of the Scholastic Aptitude Test of the College Entrance Examination Board had a higher correlation with the grades in mathematics than the entire test or the verbal section alone.

Douglass and Michaelson[6] found that the average mark in high school mathematics had a definite correlation with the average college mark in every field. The data also indicated that, in the prediction of success in elementary college mathematics, the average high school grade in mathematics and the average high school mark in all subjects are of approximately equal merit. These investigators concluded that the success of students in college mathematics cannot be predicted with any high degree of accuracy from the number of terms of mathematics taken in high school, rank on the Psychological Examination of the American Council on Education or a combination of these factors.

Similar results were obtained in investigations[7,8,9] attempting to predict a student's grade in high school algebra.

Although these studies do not agree completely on the relative effectiveness of the different factors considered for predicting a student's success in a first course in college mathematics, they do indicate that a mathematics training or placement test is the most effective single criterion. Sectioning on the basis of a mathematics training test was tried at the University of Pittsburgh. Held[10] reports that for the year he made his study failures were reduced from 21 per cent to 6 per cent by sectioning the students according to their scores on a mathematics training test. A test given at the end of the term to those students who were assigned to a course which reviewed the fundamentals indicated that most of these students had profited sufficiently from this review to register in one of the regular courses the next term. From the results of this investigation Held concluded that the use of a single criterion—the grade on a placement test in mathematics given prior to registration)was quite satisfactory.

[4] Irick, P. E., "A Study of Factors Related to Engineering Mathematics at Purdue University." Masters Thesis, 1945, Purdue University.

[5] Dichter, M. R., "Relationship between Aptitude Scores and College Grades in Mathematics and Science." *American Association of Collegiate Registrars*, Vol. 13, pp. 16–37, 1937.

[6] Douglass, H. R. and Michaelson, J. H., "The Relation of High School Mathematics to College Marks and of Other Factors to College Marks in Mathematics." *School Review*, Vol. 44, pp. 615–619, 1936.

[7] Kertes, F., "Ability Grouping in the High School." The Mathematics Teacher, Vol. 25, pp. 5–16, 1932.

[8] Orleans, J. B., "A Study of Prognosis of Probable Success in Algebra and in Geometry." The Mathematics Teacher, Vol. 28, pp. 165–180; pp. 225–246, 1934.

[9] Douglass, H. R., "The Prediction of Pupil Success in High School Mathematics." The Mathematics Teacher, Vol. 28, pp. 489–504, 1935.

[10] Held, O. C., "A College Mathematics Placement Test." *Journal of Higher Education*, Vol. 13, pp. 39–40, 1942.

Object

The studies summarized in the preceding section indicate, that of the various measurable factors considered, a mathematics training or placement test is the best single measure for predicting the success of a student in a first course in college mathematics. This naturally raises the question as to whether there is a better single measure not previously considered in the published reports on this problem. The purpose of this paper is to report on the results of a method of sectioning first term mathematics students, and to present the results which tend to show that sectioning on the basis of a mathematics training test given after the students have had a short review of the fundamentals of algebra is more effective in predicting success in a first course in college mathematics (this course consisted of topics in college algebra and plane trigonmetry) than a similar type of test given before classes began, that is, during the orientation or pre-registration period.

Such a procedure, that is, giving a test after a review of the fundamentals of a subject in order to place students in the proper course, was tried at the University of Illinois.[11] In this case the subject was trigonometry. The students were all enrolled in registration in the same course. The first five days were devoted to a review of trigonometry. A test was given after this review and the students were sectioned on the basis of their marks on this test. This procedure is similar in some respects to that used in the study to be discussed in the following sections. Unfortunately the Illinois report on sectioning after a review of the fundamentals of a subject gives no specific information as to whether sectioning in terms of a score on a test given after a review of the subject is more effective than sectioning on the basis of a similar test given before registration.

[11] Bailey, H. W., "Trigonometry in the High School." The Mathematics Teacher, Vol. 25, pp. 303–308, 1932.

Procedure

All first term engineering students who enrolled at Purdue University for the July term, 1944 were included in this study. Classes in two courses in college algebra and trigonometry, which we shall designate as M1 and M3, were scheduled at the same hours. This arrangement made it possible to change a student from one course to the other without disrupting his schedule. The two courses were different. Course M1 was at a more elementary level. In this course more time was spent on the fundamental manipulative operations. In addition the standards for passing were not as high as they were for M3. Thus we see that M1 differed from M3 in the content included and in the level of performance demanded.

Those enrolled in M1 were supposed to be the more poorly prepared students. The assignments to the two courses were made at registration. Because of certain circumstances sufficient information on many of the students was not available at the time these assignments were made. Consequently, it became evident at the end of several weeks that many of the students had been assigned to the wrong course.

Although the scores on the mathematics training test, which had been given prior to registration were now available for purposes of sectioning, it was felt that the results of another uniform test might further aid in the process of sectioning. Since the first few weeks were devoted to a review of the fundamentals of algebra through quadratic equations in both M1 and M3 it was decided to give a test at the end of this review period. Only those topics which had been reviewed were included on the test. Fifteen lessons were used for review in M3 while twenty lessons were used to review the same topics in M1.

The same test (Test 3) was given to all students during a regular class period of fifty minutes. This test required the student to solve the problem and write the answer in the space provided for it.

The test consisted of typical problems, except that all problems were very short, that is, no question involved any long sequence or chain of operations. Hence, many more questions, of this type, could be used. Further the test was mimeographed and ample space, adjacent to the problems, was left for solving the problems and also a well defined place, opposite the problem, was left for the answer. The test was graded objectively. Each problem was graded either right or wrong. The student's score was the number right. The test consisted of thirty-seven items.

The mathematics training test (Test 4) was of a similar type and the method of grading was the same. This test, test 4, consisted of questions on arithmetic and the algebra which a high school student would have covered in the three term algebra sequence. However there were very few items going beyond first year algebra as taught in high school. A total of fifty minutes was allowed for this test and it was given to the students on the day before they registered. There were forty-one items on this test.

A third test (Test 2) was given at the end of the term to all of the students in both courses. This was an achievement test and covered only those topics in algebra and trigonometry which had been studied during the term in both courses. The test consisted of eighty items and required one-hundred minutes or two class periods. This test was different from the other two tests in that it was a multiple choice test arranged for machine scoring. Since this test was constructed to measure the student's achievement on the fundamentals of the two courses, the student's grade on this test might be thought of as an objective final grade insofar as a single test can measure a student's mastery of a given course.

Data

To determine whether the test (Test 3) given after the review is better for predicting a student's success in a first course in mathematics than the mathematics training test (Test 4) we correlated the student's final grade with his score on Test 3, and the student's final grade with his score on Test 4. The degree to which Test 3 and Test 4 correlate with final grades is a measure of their relative effectiveness in predicting final grades. The difference between these correlation coefficients divided by the standard deviation of their differences is called the critical ratio. The value of this ratio enables us to state whether the difference in the correlation coefficients is significant or not, and also to calculate what the odds are that this difference is not due to chance alone.

Since it may be argued that final grades are fairly subjective we made a comparison similar to that in the preceding paragraph with regard to Test 3 and Test 4 but using the scores on Test 2 in place of final grades. These correlations will show how well each of the two tests—Test 3 and Test 4—would predict a student's score on Test 2 and consequently, be another indication of which test is the more effective for purposes of prediction

We list the data obtained. By r_{g3} is. meant the Pearson Product Moment coefficient of correlation for final grades versus scores on Test 3. Similarly, r_{23} is the correlation coefficient for scores on Test 2 versus scores on Test 3, etc.

Test 2—Objective achievement test covering the topics studied during the term.

Test 3—Test given after a review of the fundamentals of algebra through quadratic equations.

Test 4—Mathematics training test given prior to registration.

The correlation between the scores on Test 3 and Test 4 was used in computing the standard deviations of the differences of the r's. For M3 we have $r_{34}=0.664$, and for M1 we have $r_{34}=0.698$.

The above results indicate that a mathematics training test given to students after they have had a short review of the funda-

TABLE A

Course	Correlation Coefficients	Standard Deviation of the Difference of the r's	Critical Ratio	Number of Students
M3	$r_{g3}=0.613$; $r_{g4}=0.556$	0.0387	1.47	270
	$r_{23}=0.707$; $r_{24}=0.665$	0.0335	1.25	
M1	$r_{g3}=0.731$; $r_{g4}=0.575$	0.0469	3.32	163
	$r_{23}=0.709$; $r_{24}=0.599$	0.0441	2.49	

mentals of algebra correlates better with both final grades and with scores on Test 2 than does a similar test given prior to such a review. For M1 the differences in these correlation coefficients are definitely significant while for M3 they are not significant.* The differences, however, are in favor of Test 3.

When the correlation coefficient between two test scores is known, we can use the coefficient of alienation, $k=\sqrt{1-r^2}$ as a relative measure of the ability of one test to predict the students grade or mark on the second test. Thus, if $r=0.80$, then $k=0.60$. This means that for a given student if we used the mark on one test for predicting his mark on another test when the correlation coefficient for the two tests is 0.80 then there remains 60 per cent of chance in this prediction. Hence the lower the value of k the better one mark can be predicted from another. This may seem somewhat discouraging since many correlation coefficients actually obtained are not as large as 0.80. It should be observed at this point, however, that this does not mean that there would remain 60 per cent of guess if, for example, we are merely trying to predict whether a student with a certain mark on a test will pass or fail the course or make above or below a given mark on a test.

By ${}^kg3$ we mean the coefficient of alienation for ${}^rg3$, etc. The values of the different coefficients of alienation are given in Table B.

TABLE B

Course	Coefficients of Alienation	
M3	$k_{g3}=0.79$	$k_{g4}=0.93$
	$k_{23}=0.71$	$k_{24}=0.75$
M1	$k_{g3}=0.68$	$k_{g4}=0.82$
	$k_{23}=0.71$	$k_{24}=0.80$

From Table B we see that for M3 the test given after a review of the fundamentals of algebra (Test 3) is 4 per cent better than the test given prior to registration (Test 4) for predicting either a student's final grade or his score on Test 2. For M1, Test 3 is 14 per cent better than Test 4 for predicting final grades, and 9 per cent better for predicting scores on Test 2. Hence, if we wish to predict final grades on the basis of the scores on the objective achievement test, there is apparently a very definite advantage in using the results obtained from the test given after the review for those who made low marks on the test given prior to registration. That is, the test given after a review of the fundamentals is definitely better for predicting final grades for students whose preparation in mathematics is relatively weak. As a matter of fact the gain in ability to predict a student's success appears sufficient to warrant the administration of such a test, particularly since there is also some gain in ability to predict the success of those who had better preparation.

It is interesting to note that the test given after the review predicts equally well the grades on the achievement test for both M1 and M3. Such is not the case

* The selection of the fiducial (confidence) limits is, of course, arbitrary, but 95 percent is customarily taken as an acceptable fiducial probability for satisfactory significance and 99 per cent for high significance. Following the lead of Fisher we say that the differences are significant, if for a normal distribution, we have a critical ratio of 1.96 and highly significant for a critical ratio of 2.58 or higher.

for the test given prior to registration.

The results obtained from this experiment indicates that the most effective single criterion for predicting student success in a first course in college algebra and trigonometry is a mathematics training test given after a review of the fundamentals of algebra. Since such a test apparently enables us to predict with greater accuracy, its use seems justified when a knowledge of a student's probable success in a given course is needed.

The effectiveness of Test 3 for predicting purposes is relatively good when one considers that k^{12} is as high as 0.65. That is, in predicting a students final grade from his score on the test given after the review there remains 79 per cent of chance in the prediction while in predicting a student's final grade from the achievement test (Test 2) there still remains 65 per cent of chance in the prediction. It should be recalled that Test 2 was given at the end of the term and covers the essentials of algebra and trigonometry taught in the course so that this test might be considered as one measure of the student's mastery of the subject.

In schools that plan to section students in a first course in mathematics and when there is sufficient enrollment to have classes of the different courses or levels scheduled at the same hour this study suggests that the preliminary assignments should be made on the basis of a mathematics training test given prior to the beginning of classes since this is the best single criterion available. If the results obtained from this experiment are correct, then a second test given after a review of the fundamentals of algebra would be helpful in locating many of those students who had been incorrectly assigned in the beginning.

The authors recognize that the conclusions stated are only suggestive of the possible value of such a procedure and that similar studies should be made by other investigators in order to check the validity of the results obtained in this experiment.

The Integration of Trigonometry with Physical Science

By Edna E. Kramer
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A current tendency in education is the fusion of allied subjects into single courses of study. Because mathematics is so specialized in nature it will, in all probability, never be completely combined with other fields. To meet the present challenge, however, we must introduce more and more material of applied or related nature. This article is devoted to the details of one unit of such subject matter which we are using with success in our trigonometry classes at Thomas Jefferson High School.

The selection of the material in question is based on the observation that, in the past, applications of trigonometry constantly emphasized its *computational* aspects. The subject was thought of as the numerical tool of the surveyor, navigator, astronomer, and all roads seemed to lead to logarithms. College teachers have been protesting against this trend and asking for a return to former syllabi which included more *analytic* trigonometry—more difficult identities, etc, as a background for calculus and higher analysis in general.

Our own point of view is a novel one, pointing to a different deficiency in trigonometry syllabi. From the cultural or even from the practical point of view it is neither the numerical nor the analytic aspects which should concern the student most. Instead, the chief connection for the general student living in an atomic age must be with theoretical phases of *physical science*. With this in view, we are presenting the following unit to our trigonometry students at school, in addition to applications of the subject to other fields.

We start with a discussion of the manufacture of electricity. We explain that when a closed wire rotates within a magnetic field, electricity is *induced* in the wire. The meaning of this fact is made mathematical by a study of Figure I.

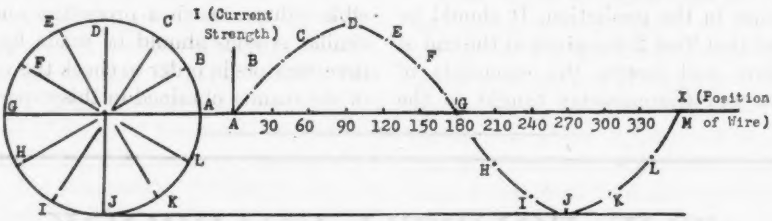


Figure I

The electricity which lights most homes is described as AC or Alternating Current. Current strength in circuits varies so rapidly that we do not notice the fluctuation. In most homes there are 60 cycles of current each second. If we represent current by I and choose a scale which gives the maximum current value as *one unit*, then it is shown in science that the following chart would represent the situation:

t (Time in Seconds)	x	I
0	0	0
—	—	1
—	—	0
—	—	−1

1/60	6° or $\frac{\pi}{30}$ radians	0
—	—	1
—	—	0
—	—	−1
2/60	12° or $\frac{2\pi}{30}$ radians	0
—	—	—
—	—	—
1 sec	360° or 2π radians	0

Note that the equations describing this chart are

$$I \sin 60x$$

$$x = 360t \text{ if } x \text{ is measured in degrees}$$
$$\text{or } x = 2\pi t \text{ if } x \text{ is measured in radians}$$

Now in science *time* is a very important element and hence it is customary to eliminate x and express I in terms of t.

Substituting in the first equation the value of x given by the third equation, we have

$$I = \sin 120\pi t$$

Exercise: What is the formula connecting I and t if there are 120 cycles per second? 70 per second?

In our work in trigonometry a comparison of the graphs of $y = 3 \sin 2x$ and $y = 5 \sin 2x$ showed that the maximum height or *amplitude* is 3 in the first case and 5 in the second. The number of cycles (or we might say *waves* of the sine graph) for one second in the illustration above is 60. The *period* is 1/60, meaning that one cycle is completed in 1/60 of a second. In the equation $I = 10 \sin 120\pi t$ the coefficient 10 is the amplitude, while the frequency is obtained by dividing 120π by 2π, giving 60. *Thus the period is the reciprocal of the frequency.*

In general, if the equation is $I = a \sin bt$, the amplitude is a, the frequency is $b/2\pi$, the period is $2\pi/b$

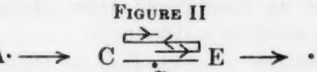
Exercises

1. *If $I = 30 \sin 200\pi t$, how many cycles per second of alternating current are there? What is the amplitude, the period?*
2. *If $I = 6 \sin 180\, \pi t$, give amplitude, frequency and period.*

The trigonometric functions are important because they have the property of *periodicity*, which is characteristic of many things in science and practical life. We have, for example, the regular succession of day and night due to the periodic rotation of the earth, the cycling of the earth in its orbit producing the periodic change in seasons. Then there is man-made periodicity as illustrated in business cycles, alternating currents, etc. In our study of trigonometry we have learned that the sine and cosine functions have the period 2π, the tangent and cotangent have the period π, etc. By adjusting the angle, e.g. considering $\sin 2x$, $\sin \frac{1}{2}x$, $\cos 4\pi x$, etc. we can adjust the size of the period at will, just as we did in the case of alternating currents above. That is why the trigonometric functions are adaptable to the description of periodic phenomena in science and every-day life.

Sound is transmitted in air (or other mediums) by oscillations of individual particles. When sound travels from A to B

FIGURE II



in Figure II, a particle at A oscillates first, then an adjoining particle, then one near that, etc. until B is reached.

Let $C\,E$ represent the range of oscillation of the particle P. If we were to plot a graph of *position* of the oscillating particle versus *time*, considering position $P = 0$, positions to the right positive, positions to the left negative, then science and more advanced mathematics show that the result would be a *sine* graph.

Since sound travels in all directions, the positions of the first set of particles to oscillate would lie on a sphere with center at A in Fig. III and a radius which varies as the particles oscillate. Successive sets of oscillating particles would be represented by larger and larger spheres which would

expand and contract rhythmically.

If a disturbance at A (Figure III) reaches B and all points of the sphere to which B belongs in one second, then there are 5 waves per second and the *frequency* is 5, while the *period* is $\frac{1}{5}$ second. The *wave-length* is the distance AS or SP or PD, etc. (Note: Sound frequencies are much higher than 5. This is an hypothetical situation for the purpose of ease in diagramming). Again, if we were to plot the varying *radius* lengths of a single sphere as oscillations take place, the result would be a *sine* curve.

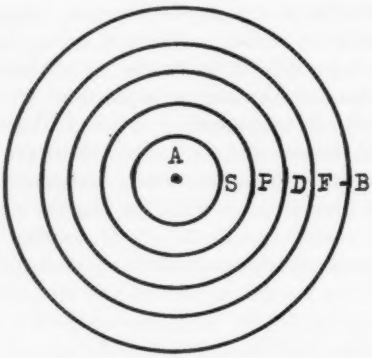


FIGURE III

In classic physics, *light* or *electromagnetic waves* are imagined to travel through an imaginary "ether" and the mode of oscillation is different from that for sound. But whether the medium is air, ether, or water (as it would be if you dropped a stone in a lake and noticed the water ripples spreading out in larger and larger circles) the particles of the medium which carry the energy are vibrating but the medium as a whole is not permanently altered or displaced.

If each wave length AS, or SP etc. in Fig. III is 2 inches long and we assume again that the disturbance starting at A reaches B in one second, then the entire disturbance will travel 10 inches in one second, that is, the distance from the source of the wave motion to the furthest point it will touch after one second is 10 inches. In mathematics we term the *distance* traveled in *one unit of time* speed or *velocity*.

If the frequency is 10 per minute and the wave length is 4 feet, velocity will be 40 ft. per minute. In general

$$\text{Velocity} = \text{Frequency} \times \text{Wave Length}$$

or

$$V = FL$$

In *sound* the frequency of a tone is called its *pitch*. Soprano is higher in pitch than bass, which means that the frequency of soprano notes is higher than that of bass. The lowest pitch notes which are audible have a frequency of 20 per second, the highest audible have a frequency of about 20000. Middle C on the piano has a frequency of 256.

In light, frequency determines *color*. Violet light has a higher frequency than blue which has a higher frequency than yellow, while red has the lowest frequency of all visible light. Invisible "light" takes the form of ultra-violet in the direction of increasing frequency and infra-red in the direction of decreasing frequency.

Exercises

1. *The velocity of light is 3×10^{10} cms. The frequency of ultra-violet light is 10^{16} per sec. What is the wave-length of this light?*

2. *The frequency of middle C on the piano is 256 (vibrations per sec). Its wave-length is $4\frac{1}{5}$ feet. Find the approximate velocity of sound (in air).*

3. *The velocity of all electromagnetic waves, including radio waves, is the same as the speed of light. (see example 1). If the length of certain long radio waves is about 3×10^5 centimeters, (this is about 3000 meters or roughly 3000 yards), find their frequency. How many kilocycles is this, if one kilocycle represents a frequency of 1000?*

4. *Explain the following statements:-*

(a) *The wave length of light varies inversely as the frequency.*

(b) *In any type of wave motion, wave length varies inversely as frequency.*

5. *Complete following statements:*

(a) *If the frequency of one sound is twice that of another its wave-length is* ————

(b) *If one color has one third the wave length of another its frequency is* ————.

6. *Which has the longer wave-length—violet or red light? blue or yellow?*

7. *The measure of the energy of any type of light or electromagnetic radiation is called its quantum.*

Planck's Law states that the size of the *quantum varies directly as the frequency.* Compare the quanta (energies) of the following:

Type of Radiation	Frequency
1. Cosmic Rays	10^{22}
2. Gamma Ray (radiation from radioactive substances)	10^{20}
3. X-rays	10^{18}
4. Ultra violet	10^{16}
5. Visible light	10^{15}
6. Infra-red rays	10^{14}
7. Radio waves	from 10^{4} to 10^{9}

8. *Fill in the blank below:*

The quantum varies ———— *as the wave length. Which have the greater wave length—cosmic rays or visible light rays? How many times greater?*

9. *In music one note is an octave higher than another when its frequency is twice as great. If the frequency of middle C is 256. what is the frequency of high C (an octave higher)? What is the frequency of an octave below middle C?*

10. *If the frequency of high C is 512, what is its period?*

11. *Give amplitude (loudness) period and frequency (pitch) of the sound described by the equation*

$$y = 0.0004 \sin 2000t$$

12. *Compare for loudness and pitch the sound described by*

$$y = 0.0001 \sin 4000t$$

with the sound given by the equation in ex. 2.

In exercise (11) we have seen that amplitude (loudness) = 0.0004 and frequency is $1000/\pi$. In general, in the equation

$$y = a \sin bt$$

a determines the *loudness* of the sound and *b* determines its *pitch*.

Another property of a musical sound is its *quality*. If a sound of the same loudness and pitch is produced by a tuning fork, violin, clarinet, it will sound different in each case. An instrument called a *phonodeik* can be used to determine the graphs of these sounds and they will appear as in Fig. IV.

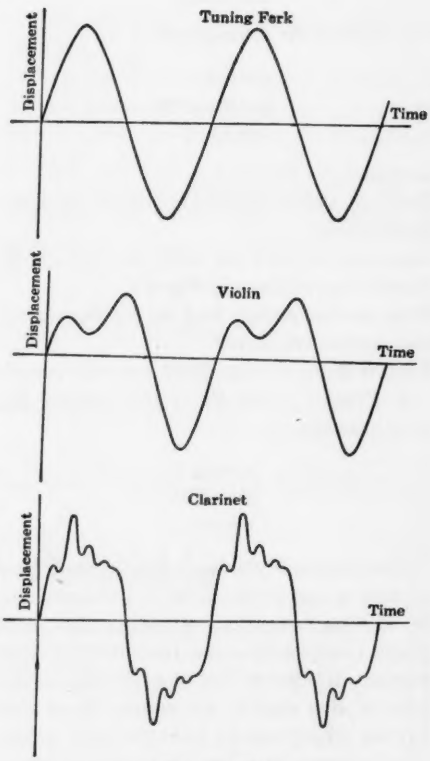


FIGURE IV

Note that periods and amplitudes are the same, but only the tuning fork graph has the appearance of a simple sine curve.

Most musical sounds are not simple sine curves, but are *composite*. The phonodeik or harmonic analyzer, as it is sometimes called, shows that composite sounds are *sums of sine curves*. For example, a tone which has the quality of a violin note of

frequency 500 can be obtained by striking simultaneously three tuning forks with frequencies 500, 1000 and 1500, and amplitudes in the ratio 6:2:1. The strongest tone (greatest amplitude = 6) is called the *fundamental*, and the other frequencies are called *overtones*. The exercises below will give some idea of the appearance of the curves representing composite musical sounds.

Exercises

1. *Plot on the same graph*

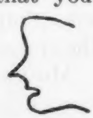
$$y=6 \sin t$$
$$y=2 \sin 2t$$
$$y=\sin 3$$

for values of t from 0 to π
Then, by adding ordinates, obtain the composite curve.
This will be $y=6 \sin t+2 \sin 2t+\sin 3t$
Note the resemblance to Fig. IV.
What are the periods and the frequencies of each component curve?
What is the frequency of the composite curve?

2. *Proceed as in Ex. 1 and answer the same questions for*

$$y=\sin t$$
$$y=\sin 3t$$
$$y=\sin 5t$$

The exercises you have just worked give an idea of one of the most important facts in all mathematics, namely that any graph (subject to a few limitations unimportant in science) can be expressed as the sum of sine curves. Of course three sine curves might not be enough—you might need a dozen or a hundred or even more. The amplitudes and frequencies for these sine curves might not be "easy" numbers. Still it is a remarkable fact that you might scribble any curve thus:



and it would nevertheless have a "sine equation." Since a curve represents a function, this is a fact very useful in practical science. It was discovered by the French mathematician Fourier who lived during the Napoleonic era. A thorough study of "Fourier Series" is made by every engineer, statistician, physicist, applied mathematician. Recently the Mathematical Association of America decided to publish books which would help students in scientific professions. The first book in the set was "Fourier Series" because that was considered the most important and useful topic.

Suppose that two wave motions are traveling in a medium at the same time in the same direction, that is, suppose there are two sounds or two radio waves or two rays of light starting at the same time in the same direction. Suppose also that the two wave motions have the same frequency (or period). Then they are described by

$$y=a \sin bt$$
$$y=a \sin bt$$

To make things simple take $a=1$, $b=1$, $c=3$, although we know that these numbers would not be so "easy" (For example, for sound $a=0.001$, $b=20000$, $c=0.003$ would be more sensible values).

The net result would be a composite sine curve

$y=4 \sin t$ (for the easy values)
or in the general case,

$$y=(a+c) \sin bt$$

In words, if two sets of waves have the same period, and are in the same *phase* (that is, start at the same time) they reinforce one another and the net result is a wave with the same period and amplitude equal to the *sum* of the two amplitudes. Thus a sound would merely become *louder*, light would be *brighter*, etc.

If, however, the waves are *not* in the same phase, we have *interference*. In the diagram (Fig. V) two waves of the same period and amplitude are in completely opposite phase. If the heavy curve is

$$y=\sin t$$

the dotted curve is

$$y = \sin(t - \pi)$$

which means that the second wave started π seconds later than the first.

If we add ordinates the net result is *zero.*

In this case:

Sound + Sound = Silence!
Light + Light = Darkness!

Usually the phases are not completely

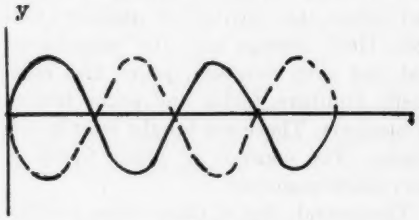


FIGURE V

opposite. When the difference in phase is slight, interference shows itself by noises in sound and radio. Teachers can show pictures of interference effects in light called Newton's rings and diffraction patterns. The following exercise will illustrate some wave patterns showing interference.

Exercises

1. *On the same graph, plot*

$$y = \sin t$$

$$y = \sin\left(t + \frac{\pi}{2}\right).$$

Add ordinates to get the sum of these two motions.

Note that the second curve is the same as $y = \cos t$ *and the combined curve is* $y = \sin t + \cos t$. *The wave motions differ in phase by* $\pi/2$ *units of time.*

2. *Combine graphically by addition of ordinates two wave motions differing in phase by* $\pi/4$. *Represent them by*

$$y = \sin t$$

$$y = \sin\left(t + \frac{\pi}{4}\right).$$

What Do We Mean by Meaning in Arithmetic?

By J. T. Johnson
Northwestern University, Evanston, Ill.

This decade has witnessed an increase in the number of articles discussing the question of meaning in arithmetic. A very good text book has appeared on ELEMENTARY ARITHMETIC: ITS MEANING AND PRACTICE.

This term, as many other terms in arithmetic, is somewhat beclouded in confusion and uncertainty. Some mean by meaning in arithmetic that a process in arithmetic gets meaning when it is applied in some practical situation. Others mean by meaning in arithmetic the nature of the processes one performs in the theoretical field of arithmetic itself. Dr. Brownell has clearly distinguished between these two and labeled them, meaning *for* and meaning *of* arithmetic, respectively.

In thinking this through still further there seems to be another meaning. It could be included under the meaning *of* arithmetic as a second subdivision under it but as it is rather inclusive, I shall for purposes of clearness and emphasis divide the meanings of arithmetic into three broad categories as follows:

I. Structural Meaning
II. Functional Meaning
III. Operational Meaning

I. Under the structural meaning would come the following;
  1. The meaning of the number symbols themselves
  2. The meaning of place value and its relation to the number system
  3. Number relations and their meanings
  4. Concepts, generalizations and principles in arithmetic

I shall illustrate each of these four for clarity.

The first one, that of the meaning of the symbols themselves, is quite well understood by elementary teachers. Most of the work in the first and second grade is concerned with the cardinal and ordinal meanings of the nine number symbols. The teacher should, however, be familiar with and appreciate the struggle the race had before they arrived at number symbols. Here belongs also the meaning of odd and even numbers, prime and composite numbers, factor and prime factors of numbers. These are taught later in the grades. The concept of prime factor is very much neglected.

The second, that of place value, is sadly neglected. I noted in the October number of the 1947 Mathematics Teacher in one of Dr. Sueltz' tests of Understanding in arithmetic that the following question was missed by 79%, 64% and 42% in grades 4, 5 and 6 respectively.

0	4	3	9	9

The meter shows the number of gallons of water used. When another gallon is used, what number will the meter show? (*A*) *4390* (*B*) *4400* (*C*) *4490* (*D*) *43991.*

When 4th, 5th and 6th graders cannot find the next larger number to a whole number any better than that there is something wrong somewhere.

There should be a systematic teaching procedure of place value in every grade from the first to the eighth. The pupils should know the value of every figure in a number starting with a one-place number in the first grade any two-place number in the second grade and so on to six-place numbers in the sixth grade on through to seven- and eight- and ten-place numbers in the seventh and eighth grades. Space allows only a few illustrations. Questions such as the following are suggestive of the meanings involved.

What does the 2 in 27 stand for? (2nd grade)

What is the meaning of the (3d grade) *0 in 309?*

Why is 903 greater than 309?

Write the smallest four- (4th grade) *place number.*

Write the largest four-place number.

Prove that you have the smallest and largest 4-place number

Arrange the following in order of their size, smallest first. (*5th grade*) *34679, 34697, 36479, 36749, 36947.*

Later in the sixth grade decimal numbers could come in many varieties.

No. 10 in the Sueltz test is an example of what should be known in the sixth grade in this connection. Better than 46% should know the answer to it. It is quoted here.

The mileage on a speedometer is shown at the right. What number will it show when another tent of a mile is added?

3	2	9	9

(A) 330.0 (B) 329.0
(C) 329.91 (D) 330.9

Then in the seventh and eighth grades rounding off numbers and its meaning could be taught with significant figures and computation with approximate numbers. This all relates to place value and its meaning in our number system.

Third in the field of number relations alone we have an important background of meanings that furnishes the basis for all of percentage later on. The work covered in number relations before percentage is taken up can be illustrated by the following:

$$\begin{array}{ccc} ? & \frac{1}{4} & 12 \\ \overline{3} & \overline{?} & \overline{12} \\ \overline{3} & \overline{\frac{1}{4}} & \overline{?} \\ \text{—} & \text{—} & \text{—} \end{array}$$

The first asks the question of finding a number that is related to 12 by the ratio of 1 to 4. The second asks, What relation has 3 to 12? The third asks, To what number is 3 related by the ratio of 1 to 4?

This then leads to the same arrangement with the ratios being of the multiple type, as;

$$\begin{array}{ccc} ? & \frac{3}{4} & 12 \\ \overline{9} & \overline{?} & \overline{12} \\ \overline{9} & \overline{\frac{3}{4}} & \overline{?} \\ \text{—} & \text{—} & \text{—} \end{array}$$

If and when these relations are well, understood in connection with many many different numbers not only will the three cases of percentage be understood but many of the principles underlying problem solving wi l be understood for problem solving and quantitative thinking in general is based largely upon seeing number relations.

Fourth in the structural meaning of arithmetic are generalizations such as these, to mention only a few; 3 and 2 is the same as 2 and 3; $2\times3=3\times2$; also, that if $5-2$ is 7 then $7-2$ is 5; and if $63\div7$ is 9 then $63\div9$ is 7. This is only a partial list. Among the principles may be mentioned the one that when both terms of a fraction are multiplied or divided by the same number the value of the fraction remains unchanged. Another principle is that known as addition by endings. For example, when you add any two numbers ending in 8 and 7 the ending figure of the sum is always 5 and so on with other endings. This principle furnishes the basis for efficient column addition. I dare say that most of us were not taught that simple but valuable principle in our youth or we would not have so many split number adders or ten-minus-one adders or adders of some other inefficient variety.

The above four illustrations will suffice to show what is meant by the structural meaning of arithmetic. It is the frame work of arithmetic, the numbers themselves and their logical interdependencies. In other words it is the meaning of arithmetic as a coherent system. The elementary school could and should place greater emphasis on this phase of meaning.

II. The functional meaning of arithmetic has been aptly called by Dr. Brownell the meaning *for* arithmetic to distin-

guish it from the meaning *of* arithmetic. It answers the question, "What is arithmetic for?" It has to with the application of the various arithmetical processes There are many teachers who think that this is the whole of meaning in arithmetic. It is true that these applications clarify and give certain meanings to the processes in arithmetic but it is a different kind of meaning from that given by the structural or the operational meanings. It consists in part of the following discoveries and learnings;

When we want to find the total or price of several items we add.

When we want to find what remains after we withdraw certain amounts of money or items, we subtract.

We also subtract when we want to find what a certain amount of price lacks in being some other amount.

We also subtract when we want to find differences in amounts of prices.

We multiply when we desire to find totals of articles or prices of the same kind.

When we find areas we multiply.

To find the price of one item knowing the price of several, we divide.

To find the average, we add the items and then divide.

This list need not be enlarged here, suffice it to say that all the uses that are given for arithmetic gives it meaning as well as motivation and this functional meaning has to be taught.

III. The third meaning in arithmetic has to do with the understanding of processes. It includes the well known term, rationalization, in arithmetic. But rationalization is only a part of this phase of meaning. Besides rationalization this phase must also include how the process is done. Some may question the *how* as belonging to a meaning phase. Here it is necessary that we do some clear thinking. The *why* cannot be given before the *how* a thing is done is known because the step which we are trying to tell the reason for in a process must be known before the reason for it can be given. An illustration from one of the following processes will be given to make this clear.

Below is listed some of the common operations in arithmetic which need rationalization;

Carrying in addition

Carrying or borrowing in subtraction

Placing of partial products in multiplication

Estimating quotient figures in division

Placing of the quotient figure in division

The long division process itself

Square root

Multiplication and division of common fractions

Handling the decimal point in division of decimals.

Let us take the simple process of carrying in addition as an illustration. When the teacher explains carrying in this example she may say;

$$\begin{array}{r} 38¢ \\ 27¢ \\ \hline \end{array}$$

8 and 7 are 15. We do not write the 15; we write only the 5 under the 7 and carry the 1 to the dimes' column, *because* the 5 in the 15¢ stands for 5¢ and the 1 stands for a dime.

Here we have an example of a very simple *how* and a simple reason *why* for the *how*. I dare say that even here there are some children who do not get the reason for the process even then. They are usually given in the same presentation for they are simple, but note that the *how* preceded the *why* nevertheless.

If we move up the scale of difficulty in the processes and take as an example, long division, let us note the tremendous increase in difficulty that the rationalization has assumed. Here is an example;

$$\begin{array}{r} 5 \\ 67\overline{)3946} \\ 335 \\ \hline 596 \end{array}$$

After having estimated from 7 into 39 and placed the 5 over he 4 as has been learned in previous lessons, shall we say the 5 is really not 5 but 50 for the 67 is really divided into the whole number, 3946. We are really subtracting 3350 from 3956 instead of 335 from 394 and have a remainder of 596 instead of 59, etc.

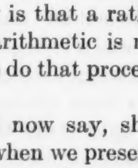
Remembering that not all pupils get the how to do each step the first time it is shown, the question arises, does giving

the reason for a step before the step is well known help or hinder the getting the how of the step?

I saw a film on long division last year at the University of Chicago. After it had been shown, many voices in the audience of teachers exclaimed that it could not be used for teaching children how to do long division. The author showing the film agreed that it was not meant for that. It is a very good film and cleverely gotten up showing what actually takes place in long division by moving and shifting about great sections of the dividend as it was diminished in the operation. It is strictly a film showing the rationalization of a process and as such it is the best so far.

But let us now imagine, if you please, an audience of adult people, fully intelligent but ignorant of how to do long division. There may be many such people in the world so it should not be difficult to imagine. Let us suppose that the film on long division were shown to them. Do you think that they would understand what it was all about? I do not think so. What I am trying to say is that a rationalization of a process in arithmetic is meaningless unless the how to do that process is understood first.

Some one will now say, shall we not rationalize then when we present new processes but merely tell the pupils that this is the way to do it because I say so? Here is where our teaching techniques come in. We must give the child a satisfaction that the way he is taught how to do a process is the correct way because he can prove or check it by observation or other means. An illustration again will make this clear. When teaching for the first time multiplication of fractions, the process is too difficult to rationalize at the time it is usually taught in the 5th grade. This is so familiar that brief mention of it here is sufficient. If $\frac{1}{2} \times \frac{1}{2}$ is to be taught illustrate with the familiar circle



and show by actual demonstration that $\frac{1}{2}$ of $\frac{1}{2}$ is actually $\frac{1}{4}$. Then follow with some more simple examples as $\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{2}$ and $\frac{1}{4}$ of $\frac{1}{2}$. Then let the pupils discover (and they like to do this) the rule that we all know. He is satisfied by the observation of the result but he has not rationalized multiplication of fractions.

It should be added here that every teacher should know how to rationalize every process in arithmetic so that when some pupil from that upper ten per cent who masters the how on first presentation asks from mental curiosity why the process is done thus and so, the teacher can give him the real answer through the rationalization. It may be above his head but she has answered his question and she can further say to him that he will fully understand when he gets a little older.

Let it be understood that I do not minimize the importance of the rôle played by rationalization. When the rationalization of a process is understood the process is better appreciated. It gives a sense of intelligent satisfaction to the learner. It becomes an integral part of his nervous system and sticks and is remembered longer. It gives a higher meaning that cannot be acquired in any other way.

But what I am trying to say here is that since rationalization of a process is not understood until the how of the process is understood and the how is not understood on first presentation by all students and since it takes a greater maturity of mind to understand the rationalization than to understand the how of a process, many teachers err in trying to rationalize every process upon first presentation before the how of the process is known. Here a sin of commission leads to a later sin of omission in this way.

Having rationalized the process when it is first taught to the teacher's satisfaction and no check having been made on the pupil's understanding of the rationalization, no further rationalization is made in later years. Here we need some research on how far pupils understand the rationalization that is given them by the teach-

ers. Teachers would be surprised to find that the process of rationalizing whole number subtraction is not understood by (I dare say) over 75% of the classes where it is taught in the third and fourth grades. I happen to have investigated this point. This is true regardless of the method used in subtraction. Tests should be forthcoming on how well the students understand the rationalization given them. Tests for meaning have been made by Dr. Suelts of Van Cortland, N. Y. and Professor Storm of DeKalb, Ill. and others. We need more studies like these on rationalization by students.

A recommended procedure would be, while these tests are being made, to teach a new process with full meaning and satisfaction to the student with rationalization only in the very simplest processes such as carrying in addition, and of course, to the upper per cent of the class that ask for it. Then in the more involved processes rationalize after the first presentation when reviews are given. What could be a better program of teaching than to bring in rationalizations of newer and higher orders as the process is reviewed in later grades. The review would then not be a rehash only, but a true review with the process seen in a new light. Research would have to lead the way showing at what mental age the various arithmetical processes could be rationalized.

Subtraction of whole numbers can be rationalized at the mental age of the normal 6th grader, that is 12 years. I do not know when the other processes of multiplication, division, common fractions, division of decimals, and percentage can be rationalized fully by pupils. The evidence is lacking. We have some evidence on this, that common fractions and percentage have never been completely rationalized in the elementary school, I mean here by "completely," only by the majority of pupils. This evidence is found in published articles on results in arithmetic by high school students and I want to call your attention here to an article in the December, 1947 issue of the MATHEMATICS TEACHER by W. J. Lyda of Morgan State College, Md. on ARITHMETIC IN THE SECONDARY SCHOOL CURRICULUM. I need not quote as all of you readers have copies. The two pages are worth reading and rereading. Please note that the high school made a poorer showing than the 7th and 8th graders. Why is this? You hear the answers given such as, "Elementary teachers do not know how to teach arithmetic," or "they teach it incorrectly" or other similar answers. I dare say that these critics would probably not have done any better with the same material. Here is my answer and I give it for what it is worth after a long experience with teachers of arithmetic in the elementary school.

The main fault lies not with the teachers but with the system. Processes in arithmetic have been rationalized, if rationalized at all, at the wrong time before the maturity of the pupils could make it stick or it was rationalized before the pupil knew the process that was being rationalized. Curriculum makers forget that arithmetic was a college subject for adults at Harvard when first taught in this country. It has been pushed downward and downward until at one time it was thought that all process work in arithmetic should be completed by the end of the sixth grade. This was a sad mistake.

I believe that the four fundamental processes with whole numbers can be rationalized by pupils in the elementary school by the end of the eighth grade. I also believe, but I am not sure (this should be tested by research) that common fractions and percentage cannot be fully rationalized until later in the high school. The article by Lyda would tend to bear this out.

To summarize on this question of rationalization:

1. Every process in arithmetic should be rationalized.
2. The how of any process should be

fully understood before rationalization.

3. The time at which rationalization should occur depends upon the difficulty of the process and the maturity of the learner.
4. The interval between the teaching the *how* of a process and its *why* may vary from one half minute to four years.
5. It may be done at the first presentation in carrying in addition.
6. Teaching the rationalization of a process before the how of the process is fully understood may confuse rather than clarify the process.
7. The length of the period between the teaching of the how and the why of an arithmetical process should be determined by research.
8. The poor showing in arithmetic problems by high school pupils and by college students is due in large part to a lack of rationalization.
9. There should be a place in the high school course (preferably in the senior year) where all arithmetic processes should be given a final review with full rationalization.
10. All teachers of arithmetic should know how to rationalize fully all processes in arithmetic.
11. The truth of any or all of the above ten statements may be disputed by educators.

In the mean time while waiting for research to help us out in furnishing tests for rationalization, teachers can do much more than is now done in teaching the structural meaning of arithmetic—place value, significant figures, approximate measurement. They can teach how to do processes better, to 100-per cent mastery on minimum essentials. They can do much with the functional meanings of arithmetic by selecting an abundance of real and practical applied problems from actual situations in the world around us.

Box 352, Southwestern Louisiana Institute, Lafayette, La.

On to Baltimore!

That very special date!

Annual Meeting of the National Council of Teachers of Mathematics March 30 and 31 and April 1 and 2, 1949 at The Lord Baltimore Hotel, Baltimore, Md. Send in your reservation now. See page 330 of the November issue of The Mathematics Teacher for hotel rates.

The Geometric Christmas Tree*

By John G. Cunningham
Northeast Senior High School, Philadelphia, Pa.

Teaching an understanding of Solid Geometry, as all teachers know, except to those gifted mathematically, is a difficult process. I have found that the construction of a model for the proposition studied is a great help to the pupil in developing mathematical understanding. In making a model the student discovers the relationship between the different parts of the figure, the reason for constructing certain lines and planes necessary to the proof, and the logical steps taken in the development of the proof.

As Christmas approached this 11 A senior-high school class, during discussion period, suggested the novel idea of trimming a Christmas tree with geometric models and calling the finished project "The Geometric Christmas Tree."

The meeting was called to order, after which the boys chose their chairman and divided themselves into the following committees.

1. Selection of Tree
2. Construction of Stand
3. Making of Spreaders and Hooks
4. Making of Additional Models
5. Trimming of Tree
6. Publicity
7. Untrimming of Tree

Everything, with the exception of the seven foot Christmas tree, was made by the pupils. It was decorated by them and stood in the front of their classroom. The models were of many colors. They were made from paper, cardboard, wood, plastic, glass, celluloid, cellophane and metal. There were models of the five regular tetraedrons, of the regular prisms, of the regular pyramids and their frustums, of the cylinder, of the cone and its frustum, of the sphere with illustrations of spherical angles, spherical polygons, spherical and polar triangles, lunes, zones, spherical sectors, segments and wedges. Solids inscribed in and circumscribed about other solids, as well as the many models from Book VI, showing the transition from plane to solid geometry, were on "The Geometric Christmas Tree." In all there were 184 models.

Due to the efforts of the publicity committee the tree was seen by practically all of the students of the school. Also the Philadelphia Evening Bulletin (the local evening newspaper), published, in their December 23 issue, a picture of the tree.

In addition to the geometric information obtained in correctly constructing the models, the students thoroughly enjoyed themselves, and received the valuable experience of working together toward the successful completion of their own project.

* See the frontispiece in this issue.

The Mathematics Teacher wishes all of its readers a very merry Christmas and a happy and prosperous New Year!

◆ THE ART OF TEACHING ◆

Colored Chalk Techniques for Basic Mathematics

By SYLVIA E. MCCURDY
Wellesley High School, Wellesley, Mass.

THE term "High School Mathematics" to the layman is the cognomen for a series of courses enabling a student to cover their college requirement in the math field. And for many years, because the efficiency of a school was so readily ascertained by college board results in those subjects, the tendency has been for pedagogues, themselves, to concentrate heavily on the techniques of teaching algebra and geometry.

However, many pupils, in spite of good teachers and equipment, came into high school, every fall, below the standard requirement for those courses and it was imperative that something be done with that group to prevent the high percentage of failures that would result if they were permitted to elect college grade subjects. In many localities those pupils were dropped into remedial arithmetic courses which were merely a repetition of subject matter and methods which have failed with them in the past, and except for a small percentage, would probably fail again. The pupil had a feeling of inferiority, of not quite making the grade, instead of the satisfaction and adventure of new horizons.

This plan has proved anything but satisfactory and the more alert mathematic departments, in the last few years, have been trying to meet this problem head on. Educators have written and publishers have printed revised basic text books, under various names and guises in such numbers that a person attempting to pick a new text is overwhelmed by their very total.

A good basic text is necessary both from the point of view of use and morale. A new book, all their own, for just their course, has achieved results with certain students even after the situation seemed almost hopeless. Rest assured, however, no matter what text is selected, a large percentage of any basic math class will never use or be able to use the instructions printed with each lesson, no matter how carefully it has been edited, because their reading comprehension has not reached the level of the standard high school text. The words "increase," "prefix," "diminish," and the like have no meaning when used in rules. The paradox of this situation is the fact that these students would resent a text "written down" to their level. Therefore a teacher must expect to interpret the instructions as a new language without appearing to explain the obvious. Unless old fundamentals are approached from a new standpoint, one-half the class will feel they know all about the subject and the other half that they never have been able to understand it so it is pointless to attempt to now.

With so many school systems considering basic mathematics a dumping course for algebra failures, it is small wonder that the students themselves have little natural pride in the course they have elected or been guided into. Therefore, it is very important to set the stage with the correct attitudes early in the course. The pupil should be made to realize that basic mathematics is to high school mathematics what the liberal arts college is to the graduate school. Since all people do not always know, at fourteen to sixteen, exactly what they want to do in life it is sometimes ad-

visable not to specialize too early. If they can be made to think of basic mathematics as the concrete foundation upon which they can build whatever type of life they chose without being held to any one pattern at this time, the word "basic" will begin to have a less undesirable connotation. They will begin to view their course, as not just an attempt to correct past deficiencies, but as a searching towards a future satisfaction that derives from selecting from "the much," "the little" that meets their personal and future needs.

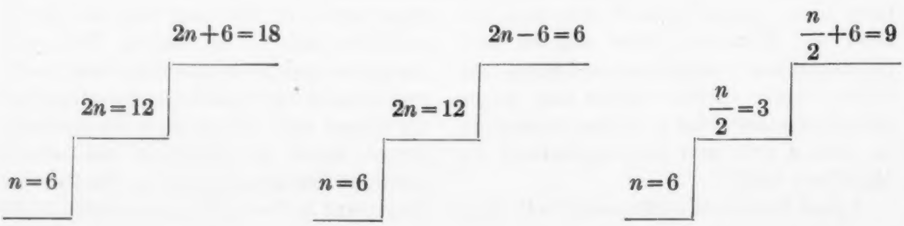
Basic classes, like college groups, are not all alike. They have their levels of efficiency from class to class and within a class. Some students have keen minds but lazy habits. This group must be challenged and lengthy explanations of techniques would bore them. Some students are industrious but lack ability. These must be given work within their capacity to accomplish with inner satisfaction. They need more help than the first group, but it must not be too obvious. These are the ones who are the most conscious of their failure because many of them are college pretential and have tried and failed. The third group, though in the minority in high school, have neither industry or ability but must be persuaded, through interest, to do something so as not to disrupt the working pattern of the class. There is always the hope that good study patterns will become a habit even with them. This gap of ability can usually be closed by maximum and minimum requirements in assignments and when the work lends itself, by extra credit for special effort even though it may mean on different levels.

Many basic texts begin with arithmetic which most classes mentally resent because they feel that they have had about all they want of that subject. The author realizes that it is advisable to have this as a background for future work. However, this can be covered quickly by the inventory method and then extra time spent on common difficulties. Also, from time to time, parts of it can be included in review quizzes. Much of this part of the book can be skimmed without danger, if the teacher continually reviews the fundamental arithmetic process before taking up a new step or introducing more advanced subjects during the year. In fact, certain pupils only understand how to solve for any letter in a formula after they have studied the solution of literal equations in algebra.

In teaching geometry to this type group, colored chalk is almost indispensable. In illustrating different types of polygons it can be used to highlight the equal parts. It makes congruency much more obvious and the reasons for certain area formulas more distinct.

In all types of graphs and algebra, colored chalk is great visual aid. It is especially helpful when used to explain sign numbers since the direction sign and the number sign are never confused in explanations because they are different colors.

The author has found it advisable and very effective when teaching algebra equations and problems to build up equations in steps and have the pupils learn to walk up and down the stairs they make themselves before they try solving the equations in the book. Some of the basic equation stairs look like this:



The use of colored chalk for each new change with "n" as a constant color helps them realize the "n" is a constant value in each set. After this the Laws of Equations seem to have more meaning. The pupils understood that equations can get more and more complicated but never so that one step at a time down the stairs won't solve them.

In problems, the pupils approach the situation by writing their own problems in English, for a given equation. They try to express an equation such as $x+8=14$ in as many ways as possible. From here, they go on to more difficult ones. This is not a new technique but seems to be effective with this type of group and more necessary for their understanding than with a college group.

A careful screening of the problem the first few days will reveal many that can be diagrammed thus:

That type of diagramming, for the first few days, leads the student to think algebraically and prevents him reverting to arithmetic in an impatient desire to arrive at the answer. The problems that do not readily lend themselves to diagramming can be taken up after the pupil has gained confidence or be left for extra credit work as a challenge to the keener pupil.

The second year of basic math, which branches into business, shop, consumer, and other fields has a wealth of visual aid opportunities readily available but unless the teacher of the first year basic math course is forever on the alert, it can be as dry as sawdust and just an uninteresting. No one yet seems to have found the perfect formula. It is such a mixture of mathematics and social service that it is never routine but certainly, more information on other teachers' experiences, and more exchange of effective techniques would make it less of a question mark.

(If a certain number) (is increased by) (8) (the result is) (14).

$x \quad + \quad 8 \quad = \quad 14$

or

(Four times a certain number) (diminished by) (6) (is equal to) (two times the number.)

$4x \quad - \quad 6 \quad = \quad 2x$

Looking Again at the Mathematical Situation*

By William Betz
Rochester, New York

Preliminary Considerations

For nearly six decades there have been almost continuous efforts to reform mathematical instruction. A comprehensive account of these efforts will serve to bring out a number of significant facts.[1] For example, from the very beginning this movement has had an international character. Moreover, in the world's leading countries it was sponsored by outstanding mathematicians and scientists. That is, the original impetus toward mathematical reform came from distinguished scholars connected with higher institutons of learning. And the issues which engaged their attention are as vital today as they were at the turn of the century. Among them may be mentioned the development of a continuous program extending from the kindergarten to the university; a persistent emphasis on such central themes as functional thinking or the study of relationships; the elimination of "inert ideas" and useless details; a genuine acquaintance with certain key concepts and methods of modern mathematics; a closer correlation of mathematics and science; the abandonment of purely mechanical drill in favor of real understanding and purposeful application; and, above all, a keen appreciation of the role of mathematics in the modern world.

We are concerned in these pages with the American phase of this epic struggle for the improvement of our mathematical curricula and our methods of teaching. And first of all, as anyone familiar with this story will readily admit, we owe an expression of deep appreciation and of lasting gratitude to that fine group of pioneering men and women who aided our cause. It was their loyalty and untiring zeal which made possible at least a partial realization of the great objectives which they had helped to formulate. They persevered in the face of the almost overwhelming obstacles to which we shall refer. For it cannot be denied that the American reform movement in mathematics has been drastically affected by two diametrically opposite influences. Future historians will find it hard to explain the conflict that has been darkening the mathematical scene during this period of world turmoil.

In the first place, there is no secret whatever about the spectacular, ever-increasing dependence of the modern world on the tools furnished by mathematics. Such fields as science, industry, technology, and economic research would soon collapse without the guiding hand of the mathematician. Experts have asserted that if, in spite of humanity's passionate desire for peace, a third world war should become a dreaded reality, it will be a *mathematical* war. Can anyone suggest a school enterprise surpassing mathematics from the standpoint of all-pervading significance? Or is there another subject-matter field of such cosmic scope, even in its most elementary ranges? In other words, our faith in the educational importance of mathematics—the prime mover of all mathematical reform—has a tremendously substantial foundation.

It should therefore be a source of acute surprise that in the United States alone,—technically the most advanced nation—, there should have arisen a veritable crusade against a proper emphasis on mathe-

* Based on addresses delivered at the Institute for Teachers of Mathematics, University of Louisville, June 18–July 9, 1948.

[1] Valuable information concerning the early phases of the reform movement in mathematics may be found in the early volumes of The Mathematics Teacher and of *School Science and Mathematics*. The further progress of these efforts is reflected or described in such publications as the *Yearbooks of the National Council of Teachers of Mathematics*, and in the standard treatises on the teaching of mathematics.

matical instruction in the schools. For many years the oldest and most indispensable of all the sciences has been under severe attack, while at the same time nearly all attempts at a thoroughgoing, universal readjustment of the mathematical program were regularly frustrated. It is a sober fact that our educational policymakers have been dominated by a chronic phobia against any type of mathematics extending beyond the range of "grocery store arithmetic." Under the impact of this attitude, amazing things have already happened. Thus, fractions and decimals have taken on the character of "higher mathematics," to be postponed indefinitely. Algebra and geometry are being reserved for "the few" who are bold enough to desire an adequate preparation for their life work. The rank and file, "the other 85%," so we are told, can get along with a menu of "life situations," a sort of "mathematics without mathematics." It will be observed that this development is on a par with that of English without grammar and literature, social studies minus history, and science without measurement. A new type of illiteracy is thus being cultivated. "Social utility" has become the all-controlling watchword. As an essential and permanent element in any educational program involving genuine literacy, as well as *cultural* training and orientation, mathematics is being assigned to the scrapheap in our secondary schools and colleges. Once again, it is a case of "killing the goose that lays the golden eggs."

In reality, this deplorable attitude has had the effect of sabotaging six decades of mathematical reform and of cancelling much of the progress that had already been achieved. It seems clear that this situation cannot be allowed to go on indefinitely, if we are really interested in the welfare of our young people and in the safety of our nation. What can be done about it? Thus far, the many suggestions and protests that have come from individual teachers and from mathematical associations have usually been ignored by those in control of our educational policies. The case must therefore be appealed, on a large scale, to the American parent and taxpayer who is still largely unaware of the actual state of affairs. It seems that only a campaign of persistent publicity will bring about the necessary reorientation. In certain communities, this battle for educational sanity has already begun.[2] It must go on until the victory is won.

In the light of the foregoing considerations, we are now confronted by a dual task, if we would teach mathematics into its rightful place. *First,* we must specify, and then seek to eliminate, the obstructions that are preventing the proper reorganization of mathematical instruction. *Second,* we must restate, again and again, those ideals, guiding principles, and basic tenets which are anchored on the painstaking labor of responsible mathematical leaders and committees, and which have been subjected to the crucial test of long-continued classroom experience.

In the present paper we shall address ourselves to the first of these tasks. An enumeration of the obstacles to mathematical reform is not a grateful task. In these pages it is not undertaken in a spirit of wanton condemnation, nor does it imply a lack of genuine appreciation of the many fine things that have been accomplished in our schools. Mass education will always remain a most complex problem. There are no "easy" solutions for that problem.

I. Wrong Theories and Their Grave Consequences

The observations submitted below are based on extensive classroom experience. Moreover, they reflect the reactions of a large group of teachers with whom the writer became associated in connection

[2] An indication of this struggle is that of public, severe progests, in one of our largest cities, against the educational policies prevailing in that community. (See the *Pathfinder*, July 28, 1948.)

with curriculum planning, with summer courses, institutes, workshops, and the like. Among the difficulties or obstructions they were encountering they regularly pointed out such factors as those discussed in the pages that follow.

1. Whither Education? In his well-known Inglis address, delivered years ago at Harvard University, Professor Dewey discussed our obvious educational confusion which he then attributed to "aimlessness."[3] That his comments did not remedy the situation, he himself seemed to realize when he wrote as follows in a more recent magazine article:

"We agree that we are uncertain as to where we are going and where we want to go, and why we are doing what we do."[4]

Coming from the man who for two generations has been regarded as the chief fountainhead of our educational ideals and policies, such a statement should have caused profound dismay in the educational world. For this declaration implies that the nation's most important enterprise is being carried on in a fog of uncertainty. But nothing of the sort happened. Apparently, we no longer care where we are going, so long as we seem to be moving.

In reality, all this occasions no surprise to experienced classroom teachers. They have long known that the present chaos is the inevitable consequence of an orientation which they have deplored from the beginning. The only element of surprise in this situation is the nationwide indifference to what is happening in our schools

Our dominant philosophy is known as pragmatic instrumentalism. But pragmatism, we are told, has only one fixed principle, namely, that *there are no fixed principles*. That almost tells the whole story. The uncritical adoption of the pragmatic approach has led from one blunder to another. It has involved the rejection of traditional values and of race experience. No longer should one refer to the "eternal verities," for they are illusions. Ideas are mere "instruments" to be used solely in solving personal or social problems of immediate interest. Truth is not inherent in ideas. Instead, an idea becomes true if it "works." There is nothing permanent. The world is what we make it. The curriculum is a changing affair, based on "novelly developing situations." It "emerges" from day to day, being concerned entirely with momentary interests. Nothing must be "fixed in advance." The child's personal "felt needs" are the controlling factor in the school. And the life situations that confront him represent his personal curriculum. Hence there can be no standards of achievement. Personal "growth" alone is significant.

This catalogue of aberrations could be continued indefinitely. Rootlessness, which Louis Adamic describes as the "American disease," has thus invaded education from top to bottom. Whither education?

To be sure, a not inconsiderable literature of protest against these trends has sprung up. By slow degrees, a reaction has set in against both pragmatism and experimentalism.[5] It will take on momentum only as all teachers acquaint themselves with the factors that have brought on the present crisis.

Perhaps the most authoritative and most informative study of the breakdown in education is that of Professor I. L. Kandel.[6] It deserves the most careful attention of every teacher.

What remedies, if any, can be suggested for the alleviation of our educational troubles?

Turning again to Professor Dewey, we

[3] Dewey, John, *The Way Out of Educational Confusion*, Harvard University Press, 1931.

[4] From "Challenge to Liberal Thought," by John Dewey, in *Fortune*, August, 1944, p. 155.

[5] See, for example, Sonderquist, Harold, "Philosophy of Education," in *Review of Educational Research*, June, 1945, pp. 196–204. This report contains 54 references.

[6] Kandel, I. L., *The Cult of Uncertainty*, New York, The Macmillan Company, 1943.

find that in the final paragraph of his treatise, *Experience and Education* (1938), he offers these words of wisdom:

"What we want and need is education pure and simple, and we shall make surer and faster progress when we devote ourselves to finding out just what education is and what conditions have to be satisfied in order that education may be a reality and not a name or a slogan."

Most admirable,—but just what is "education pure and simple," in Dewey's pragmatic setting?

Is it not clear that we must, first of all, give up the disastrous theories that have led us astray? In addition, as Professor Kandel suggests, we must develop "calmness and strength, a sense of social stability, and a *feeling of membership in a common culture.*"

Perhaps we have now explained why mathematics has aroused the passionate ire of so many "educators." Teachers of mathematics should entertain a sense of great satisfaction that their subject is the very antithesis of the doctrines which Professor Kandel has characterized as a "retreat from reason." Mathematics is the very embodiment of permanence. It does not "emerge" from day to day. It is rooted in a very deep soil. Long after the vagaries of this age shall have been swept away, mathematics will continue to be one of the great unifiers of the race, because of its global significance and its cosmic perspective.

2. Unworkable Educational Theories. The frenzied enthusiasm which once greeted the involved body of educational doctrines known has the "new education" has been followed by a decided "cooling-off period." For when the schools applied to these doctrines the test of "negative pragmatism," to use an apt phrase due to Professor Hocking, it became clear as crystal that many of them simply would not "work." For example, the idea that a group of forty or more unassorted youngsters, under conditions of mass education, can evolve their own "integrated" curricula on the basis of personal, direct "experience," is just too naive for words. Again, the notion that Johnny and Mary, unburdened by real knowledge, can be made into bold experimentalists who, by applying the "scientific method," will bravely set out to "reconstruct the universe," is likewise tragically amusing.

What, then, is the present status of "Deweyism" in our schools? To understand its real nature, we must carefully distinguish, as Meiklejohn has suggested, between the "early" Dewey, the author of *School and Society*, the brave protagonist of necessary reforms, and Dewey the philospher of a later period.[7] The immense body of Dewey's philosophic writings can be evaluated only by trained specialists. Today this enormous structure is no longer regarded as virtually sacrosanct. Thus, the pragmatic theory of knowledge, when critically examined by so distinguished a philosopher as Professor Blanshard of Yale University, revealed fundamental misconceptions and incorrigible inconsistencies.[8] Similar remarks apply to other aspects of Dewey's philosophic position.

In like manner, the serious consequences of Kilpatrick's gospel of "change" and of Thorndike's "laws" of learning, as commonly interpreted, are now being recognized more widely.

Finally, the fact that some of our most

[7] For critical reviews of our prevailing educational theories and philosophies, see, for example, Schilpp, Paul A., editor, *The Philosophy of John Dewey*, Northwestern University, 1939; Kandel, I. L., *Conflicting Theories of Education*, The Macmillan Company, 1938; Breed, Frederick S., *Education and the New Realism*, The Macmillan Company, 1939; Meiklejohn, Alexander, *Education Between Two Worlds*, Harper and Brothers, 1942.

[8] Blanshard, Brand, *The Nature of Thought*, The Macmillan Company, 1940, Volume One, Chapter X, "Pragmatism and Thought," pp. 341 ff. This critical chapter from Professor Blanshard's monumental treatise, together with Meiklejohn's penetrating study (op. cit.), will effectively orient teachers as to the basic nature of "Deweyism."

popular educational theories and policies are mutually incompatible or contradictory, can no longer be glossed over. Thus, the incurable conflict between Kilpatrick's glorification of the "individual" and Dewey's passionate emphasis on "social" objectives really caused the collapse of Progressive Education. In like manner, one cannot consistently espouse "evolving" curricula which "emerge" or change from day to day, and at the same time impose at the end of the school year a set of uniform, standardized tests presupposing a fixed program.

Teachers have been in anguish so long over the confusing array of current educational slogans and miracle formulas that nothing but a basic redirection can be accepted as satisfactory.

3. The Growing Curricular Anarchy. The preparation of a sound and really up-to-date curriculum represents an extremely technical task which presupposes wide experience and dependable scholarship. Only too often that fact was ignored during the recent period of nationwide curriculum revision when every town and hamlet felt the urge to develop its own brand of streamlined curricula. The results were spectacular in the extreme. Nearly 100,000 courses of study are now cluttering the shelves of our curriculum "laboratories." As was to be expected, only a minority were found to have real merit.[9] And the end is not yet.

As always, "too many cooks spoil the broth." Purely local curricula tend to reflect so many divergent points of view that long-established guiding principles are bound to fade into oblivion. To such circumstances we must therefore ascribe the prevailing multiplicity of objectives which Professor Knight has characterized in these words:

"More than fifteen hundred social aims of the study of English, more than three hundred aims of arithmetic in the first six grades, and more than eight hundred generalized aims of the social studies have been listed here and there in courses of study and in special studies. In one course for the social studies in the seventh grade appeared one hundred thirty-five aims; in another subject more than eighty aims were found; the objectives of a junior-high-school course were so numerous as to require many pages merely for their listing."[10]

How can we find our way out of this curricular anarchy?

At least three major considerations seem pertinent at this point. *First,* as Walter Lippmann has so emphatically stated, this situation is rapidly leading to the cultural disintegration of our national life.[11] Not local and sometimes trivial preferences, but relatively permanent and universally important backgrounds should constitute the heart of our curricula. *Second,* in each subject-matter field it is perfectly possible to specify a check list of those essential categories which together constitute literacy and competence in that field.[12] *Third,* local curriculum enterprises should try to be in harmony, as far as possible, with the programs resulting from the work of nationally endorsed committees.

4. "Individual Needs and Interests." It may be taken for granted that no two human beings have ever been exactly alike. The fact of "individual differences"

[9] See, Bruner, Herbert B. (and others), *What Our Schools are Teaching,* Bureau of Publications, Teachers College, Columbia University, 1941, pp. 25 ff.

[10] Knight, Edgar W., *Progress and Educational Perspective,* The Macmillan Company, 1942, p. 126.

[11] See, Lippmann, Walter, "Education vs. Western Civilization," in *The American Scholar,* Spring 1941, Vol. 10, No. 2.

[12] See, for example, the Check List for functional competence in mathematics, in "The Second Report of the Commission on Post-War Plans," THE MATHEMATICS TEACHER, May, 1945, pp. 197–198; also, the revised list, in the "Guidance Pamphlet in Mathematics for High School Students," Final Report of the same Commission, THE MATHEMATICS TEACHER, November, 1947.

requires no validation. Degrees of interest, capacities, learning rates, and the like, vary considerably in the average classroom. Hence any system of mass education inevitably faces serious difficulties, whether it attempts to enforce uniform schedules for all but exceptional pupils, or gives up definite goals in favor of "personalized" approaches. The latter alternative is now in vogue in many school systems.

The chief exponent of this policy of "adaptation," in extreme form, has been Professor Kilpatrick. Even during the war years, which called for a supreme solidarity of training, he still maintained his position. To quote,

"If the pupils of this age are to live, we can say, apparently at once: *there should be no separate subjects to learn*, people don't live that way. *There should be few if any assignments as such to learn* . . . people don't live that way. . . . *There should be no fixed-in-advance curriculum*, life does not so come."[13]

Of course, as most wage earners soon discover, life *DOES* so come. The logical consequence of Professor Kilpatrick's doctrine would be a special curriculum for every child. This would be the ultimate effect of policies initiated years ago by President Eliot when he launched the "elective system." The assumptions underlying this movement have been examined anew and discussed critically by the Commission on Philosophy in American Education. To quote,

Regarding the equality of subjects, (the first assumption), it is not true that subjects are of equal disciplinary value. It is not true that as instruments for understanding the world they are of even approximately equal value. . . . It is obvious that in the understanding of nature, for example, physics is more important than meteorology. . . . Indeed in every field there are studies that deal with essential principles and others that deal with applications and incidentals. To introduce democratic notions of equality here, to suppose that because two subjects both appear in the catalogue with equal hours and credits, or are taught by equally devoted exponents, they are therefore of about equal significance as factors in education is no doubt natural enough for openhearted youth. *It is nonsense, and costly nonsense, nevertheless.* . . . [Another] assumption of the elective system is that the student is himself competent to fix the pattern of his education. The difficulty with this assumption is that *it attributes to the student at the beginning a wisdom that comes to him only at the end, if indeed even then.* . . . Such insight is not instinctive; it is not there at the start; and *to assume this is an expensive error.* . . . As W. P. Montague has said, "to leave a child free to study any subject or none is simply to deprive him of his social inheritance. He . . . is intellectually and culturally naked. . . . Why should we expect him in the name of 'individuality' and 'self-realization' to repeat all that the race has learned by generations of trial and error?"[14]

It might be argued here, by way of reply, that the schools have set up an elaborate guidance program to aid the pupil in finding his way through the maze of several hundred school subjects. There is no doubt whatever that this service can and should be of great benefit to every boy and girl in the schools. However, the teachers feel that, only too often, "guidance" consists essentially in locating for Johnny the easiest "non-failure" route. This means that a warning sign constantly hangs over the "apt-to-fail" subjects, and especially over mathematics.

II. Destructive Administrative Policies

1. Administrative Blunders. One wrong step is often followed by another. Thus, the doctrine of extreme individualism necessarily involves the fragmentation of our curricula and the abandonment of grade standards of achievement. It is then but a short step to the policy of automatic

[13] *Bulletin, Association of American Colleges*, March, 1943, pp. 37 ff.

[14] *Philosophy in American Education*, a Report of the Commission on the Function of Philosophy in American Education, Harper and Brothers, 1945, pp. 89 ff. This report, prepared by five specially chosen members of the American Philosophical Association, offers a detailed study of the present state of philosophy and its potential role, and includes a valuable commentary on the present educational situation.

promotion, which has already been sanctioned on a wide front. When school administrators resort to that futile escape mechanism, the situation soon becomes catastrophic. Pupils are only human. As soon as they realize that, whether they work or not, they can still move forward from grade to grade, the teachers face an impossible problem. Mathematics is a cumulative enterprise. Nothing will ever change that. When the average pupil no longer has mastery of anything whatsoever, at any grade level, there is no miracle technique on earth that can remedy such conditions.

Most emphatically, there is no excuse whatever for such policies. The wholesale retardation of pupils is totally unnecessary. Aside from an insistence on really adequate instruction, it can be prevented by the simple and comparatively inexpensive plan of assigning at least one competent remedial teacher to the primary grades and to the intermediate grades. With proper provision for the prompt correction of individual difficulties, and for the maintenance of reasonable standards, the school does not invite the chaos that has now overtaken so many classrooms.

2. The Tragedy of Postponement. On a par with the corrosive effect of automatic promotion is the equally disastrous practice of postponing or "stepping up" entire blocks of work that can readily be mastered by any average pupil under really competent instruction. In the field of arithmetic, a controversy over this policy has been raging for years, ever since a small body of self-constituted investigators recommended a decided postponement of arithmetical training in the grades. In spite of all protests and critiques from authoritative judges, and in the face of the most positive evidence to the contrary, not a few administrators, as well as publishers and authors, soon gave their support to the dubious findings announced so confidently. The uncritical alacrity with which these revolutionary pronouncements were endorsed can only be explained on the basis of the total situation described above. Postponement represents another convenient escape mechanism. That it does not solve a single problem, but ultimately makes things much worse, should now be apparent to the most innocent observer. The "law of readiness" has been grievously misinterpreted. Maturation is not an automatic, purely chronological process. It is now accepted that children are *made ready* for reading. In exactly the same way, they are *made ready* for systematic number work.

Even a moment's reflection should make it clear that when the period of infancy is unnecessarily prolonged by two or more years, the entire educational edifice—the work of generations—begins to crumble. Without a proper foundation, the junior high school program is wrecked, the high school spends half its time in the vain attempt to supply the "missing links," and the college assumes the functions of the high school. These things are actually happening, all along the line, and are taking on the character of a national calamity.

Concerning the long-range dangers of postponement, the Joint Commission Report contains these warning statements:

"To postpone until college years basic preparatory studies that experience has convincingly shown can profitably be pursued in the secondary school, *gravely handicaps the pupil in his later effort to make a program of real collegiate studies. The doctrine of 'postponement,' like the doctrine of 'incidental learning,' however alluring to the short-sighted person and however valid in certain subjects, is indefensible in the case of mathematics.* The subject is so extensive and so difficult, requiring systematic and protracted study, as to be unsuitable for the general application of either of these doctrines."[15]

[15] "The Place of Mathematics in Secondary Education," the Final Report of the Joint Commission, in the ***Fifteenth Yearbook of the National Council of Teachers of Mathematics***, 1940, p. 44.

The economic consequences of postponement are truly terrifying. Or is it a small matter that millions of our young people are thus being deprived, through lack of time and dwindling financial support, of the vocational or professional training they really need or desire?

But the policy of postponement is even more serious from a standpoint of national efficiency and security. Let anyone who may doubt this study carefully that incisive document entitled *Manpower for Research,* vol. IV of the Steelman Report to the President.[16] Let him peruse the fact that it takes many years to train a research scientist, and that we have reached a low ebb in basic areas of scientific research. Today, mathematics and science are our crucial weapons in the desperate struggle for national security. By all that is sacred to us, let the schools stop the suicidal trend that is blunting these weapons!

Teachers have long known that *the policies described above do not eventuate in real kindness to the child. On the contrary, they are extremely cruel. The world does not offer a penny for a million WRONG answers. Its customary reaction to incompetence is a closed door.*

3. The Multiplicity of School Organizations. One often hears of the "dual" educational system of Europe, while our system is said to be "unitary." In the secondary field this is certainly a misconception. Thus, in a recent year, according to the United States Office of Education, we had as many as 29 types of organization. In addition to the traditional 8-4 plan, we now have 6-3-3, 6-6, 7-5, 6-4-4, and other plans.

This fact has been a source of great confusion and of extreme vexation to our curriculum committees. How can one build a satisfactory program for the ninth grade, for example, when the actual status of that grade is so completely undefined? We cannot effect a proper continuity of our educational enterprises, nor can we do away with the compartment system in mathematics, so long as we favor such a great diversity of organizational plans.

III. Learning and Teaching

1. The Mechanization of the Learning Process. Mass education tends to favor mechanical methods of teaching. That appears to be "the easy way out." But a democracy absolutely presupposes an intelligent citizenry capable of thinking. A generation raised on push-button reflexes will not measure up to the vexing issues of our time. Nevertheless, the connectionist psychology of learning, in the form given it by Thorndike, has been in control for many years. It has even been eulogized as the great panacea of learning.[17] Perhaps that was to have been expected in an age which glorifies mechanism in every department of living. However, it will remain a mystery how the schools could fall for the doctrine of "felt needs" and the elimination of standards, and *at the same time* give enthusiastic support to the absolutely opposite, mechanized program of the "measurement movement."

Now Thorndike's famous "laws" of learning, as originally stated, were based very largely on observed facts of *animal* learning. It did not seem to make much difference that children, after all, are not rats. Gradually, the flaws of Thorndike's mechanistic system became painfully obvious. In mathematics, as Brownell pointed out so clearly, the mechanistic "bond theory" virtually predetermined children to failure.[18] When Thorndike

[16] "Manpower for Research," Volume Four of *Science and Public Policy,* A Report to the President, by John R. Steelman, Chairman, October, 1947. (Reprints may be obtained from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.; price, 35¢.)

[17] See, for example, "The Psychology of Learning," Part II of the *Forty-first Yearbook of the National Society for the Study of Education,* 1942, p. 137.

[18] Brownell, William A., "Psychological Considerations in the Learning and the Teaching of Arithmetic," in the *Tenth Yearbook of the National Council of Teachers of Mathematics,* p. 6.

eventually saw fit to turn his attention to *human* learning, probably under the impact of diametrically opposite doctrines like those of Wheeler, he was forced to modify his "laws." In particular, the notorious Law of Exercise was now given the following, completely altered form:

"*Repetition of a connection* in the sense of the mere sequence of the two things in time *has very, very little power, perhaps none, as a cause of learning.* Belonging is necessary."[19]

In the meantime, however, the damage had been done. Endless mechanical drill, aided by countless workbooks and similar devices, was considered necessary to "stamp in" the multitude of "bonds," "skills," and other "elements" into which the curriculum had been dissected. As a result, we have had an era of *muscular* mathematics, devoid of real meaning and significant applications. But this atomic theory of learning never "worked," and for a very obvious reason! It remains true, as Dewey said so clearly, years ago, in a classic statement, that

"Practical skill, modes of effective technique, can be intelligently, non-mechanically *used* only when *intelligence* has played a part in their *acquisition.*"[20]

Clearly, a reconsideration of the learning process has long been overdue. It may take many years to undo the harm that has resulted from speculative theories of learning divorced from common sense and from the corrective influence of classroom experience.

2. The Absence of Planned Controls. The pragmatic "educator" constantly glorifies and invokes the "scientific method." But he completely ignores one of the most basic rules of science, namely, never to claim a scientific principle or discovery to have been established until it has been validated by numerous experts beyond all reasonable doubt. The medical profession no longer depends on purely conjectural remedies or treatments. Nor do we endanger the physical welfare of children by reckless experimentation.

But in the mental realm these precautions have been flagrantly set aside. We have forgotten that education is a one-way road, and that wrong procedures may wreck a child's mental and spiritual life beyond repair. Theorists without experience have regularly invaded the educational arena with totally untried miracle formulas. The time is at hand when public opinion should at last stop this, very much as it has stopped dubious health experiments.

Scientists work out their new ideas in research laboratories. "Industrial research" has almost become a major industry. In 1940, there were more than 2200 industrial research laboratories in the United States, with a personnel of over 70,000, and with an annual budget now estimated at half a billion dollars.

Is it too visionary to ask for a similar plan in education? In isolation, the average teacher cannot possibly cope with the countless educational problems that demand attention today. But if we had several hundred cooperating laboratory schools of high quality, with a nation-wide, coordinated program of research, and with planned controls and adequate methods of evaluation, the picture would soon change. *These schools should be manned exclusively by a corps of master teachers whose scholarship, training, and experience should compare favorably with the qualifications of experts in other fields of research.* Only then shall we finally leave behind us, once for all, the era of mere guesses or vague educational speculations.

3. The Preparation of Teachers. It is an old saying that "as is the teacher, so is the school." Fine buildings are a great asset, but without the life-giving quality that comes from good teaching, they are mere "window dressing." That primary fact, too, has been overlooked, in favor of almost every other consideration. It seems

[19] Thorndike, E. L., *Human Learning,* Appleton, 1931, p. 28.

[20] Dewey, John, *How We Think,* D. C. Heath and Company, 1910, p. 52.

to be true, according to reliable reports, that a large percentage of our teachers do not possess adequate professional qualifications.[21] The recent exodus from the teaching profession has made the situation most critical. Unless corrective measures of a national character are initiated very soon, the schools will find themselves in a well-nigh hopeless condition.

It is well to remind our educational policymakers, in this connection, that unremitting attempts have been made by all the authoritative mathematical committees to improve the training of teachers of mathematics. Attention should be directed again to Professor Archibald's extremely thorough study of this problem in the Report of the National Committee on Mathematical Requirements (pp. 429–508). The Joint Commission devoted an entire chapter of its Report to this crucial subject (op. cit., pp. 187–203). Likewise, the Second Report of the Commission on Post-War Plans rehearsed this matter in nine of its thirty-four "theses" (op. cit., pp. 215–220). When and if this total body of suggestions and recommendations is at last taken seriously, a better day will dawn in the status of mathematics.

[21] The prewar studies pertaining to the training and the certification of high school teachers, such as those of Bachman and of Hutson, certainly proved that "the only solid foundation of an educational system, the qualification of teachers, has virtually been neglected, and has been left unstandardized and chaotic." See, for example, "The Education of Teachers," by Embree, Edwin R., in *The American Scholar*, Autumn 1939, vol. 8, No. 4, pp. 422 ff. Since the war, a veritable catastrophe has overtaken the American teaching profession. Truly alarming facts are presented in Fine, Benjamin, *Our Children are Cheated—The Crisis in American Education*, Henry Holt and Company, 1947. It is shown that "since 1941 more than 350,000 teachers have left the classroom," that in one state "only 2 per cent of its present teachers were in the system in 1940–41," that "in some states 30 per cent or more of the teachers are on emergency or substandard licenses," and so on. A vigorous reaction against this situation has begun. Thus, the recent National Conference on the Education of Teachers, at Bowling Green, Ohio, State University, symbolized the profound realization that we need a different program in this field (see *NEA Journal*, September, 1948, p. 344).

A Final Word

This recital of our most pressing difficulties has not been a pleasant one. However, it has aimed solely at the correction of preventable flaws. Where there is so much light, the shadows are much more noticeable.

Perhaps we have made it clear why mathematics has been such a source of irritation to our educational policymakers, and why the educational doctrines and policies which we have discussed have been directed so largely against mathematics. Even if the teaching of mathematics were absolutely perfect, which is very far from being the case, it would still be under attack. For the subject does not fit into the scheme of education described above. It cannot be derived from "personal experience" alone; it requires honest work and cumulative mastery; it cannot be "adapted" indefinitely to "individual needs and interests." We cannot have a separate multiplication table for every child. And so long as the laws of thinking remain the same, the basic principles and facts of mathematics will not change. The age-old science of number and form cannot be twisted or cajoled into a different channel without ceasing to be "mathematics."

The hopeful element in this picture is that a reaction is setting in against the prevailing "cult of uncertainty." The nation's schools cannot operate forever in a dense fog. A return to sanity is bound to come. The present world turmoil is inexorably exposing erroneous trends and weak foundations. In this impending redirection the teachers of mathematics can and should play a vital role, for they are concerned with a subject whose very essence is truth, order, and permanence.

◆ AIDS TO TEACHING ◆

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BOOKLETS

B. 15—*One Hundred Problems in Consumer Credit*

Pollak Foundation for Economic Research; Jeffrey, New Hampshire.
Booklet; $5\frac{1}{2}'' \times 7\frac{3}{4}''$, 56 pages; \$.10

Description: This booklet, compiled by Charles H. Mergendahl and LeBaron R. Foster, was printed in 1945 for the purpose of helping people to make loans and time purchases more intelligently. The pamphlet first explains the basic terms and concepts involved in purchasing on credit and then suggests methods of solving the problems which arise. Part I of the booklet is primarily concerned with problems which may be solved by means of the simple interest formula and by constant ratio methods. Part II is designed for use by students of college algebra, advanced business mathematics, and the mathematics of finance. The exercises in this section may be solved by constant ratio methods. Mention is made of scientific actuarial solutions.

Appraisal: In the present age of budgets and consumer credit, the purchaser must be equipped to solve basic financial problems. This need has been widely recognized as evidenced by recent attempts to socialize the high school curriculum—especially on the junior high school level. The problems in this booklet are not only interesting but practical as well; they were selected from information gathered by Better Business Bureaus, Legal Aid Societies, and from advertisements dealing with consumer credit. Another favorable feature is that its problems are graded according to difficulty in the following subdivisions: grammar and junior high school, high school, and college level. It is advantageous to supply each student with a copy. If the price is prohibitive, a single copy may be purchased and reproduced in part. The publishers have indicated that any part of the booklet may be reproduced without permission. (Reviewed by Bernard Singer, Boston, Massachusetts.)

EQUIPMENT

E. 7—*Multiplication Game*

I. D. Fogelson; 5520 South Shore Drive; Chicago 37, Illinois.
Arithmetic game; $9\frac{1}{2}'' \times 12\frac{1}{2}''$; 2 cardboard sheets; 50¢.

Description: This game consists of one piece of cardboard which can be cut into 144 numbers representing the products from a multiplication table and another, slightly larger, on which these products can be placed by the child playing the game. The numbers are drawn after they are cut up and placed in their proper square until the table is completed. The cut numbers are five-eighths of an inch square and they are placed on squares each three-quarters of an inch square.

Appraisal: The motivation of this simple game should appeal to elementary pupils and give them one more way of learning their multiplication facts. The cardboard upon which the game is printed is of medium durability and may not last many years. This is another of Miss Fogelson's good ideas which might be purchased

for each school or school system to show teachers how they can have their pupils invent and manufacture simple games.

FILMS

F. 25—*How to Change Fractions*
F. 26—*How to Divide Fractions*
F. 27—*How to Multiply Fractions*

Johnson Hunt Productions; 1133 North Highland Avenue, Hollywood 38, California.
16 mm. sound film; 1 reel; black and white —\$45, color—\$85; 1947

Content of F. 25: This film is primarily concerned with the reasoning behind the methods of finding equal fractions, and an explanation of why the new fraction is of equal value. Segments of a disk are usually used to represent fractional parts. A short review is first given to remind the pupil of the meaning of fractions and terms of a fraction. Familiar objects, such as a cake and a pie, are used to illustrate the meaning of equal fractions. A disk is used to explain how a fraction is changed to a fraction with higher terms by multiplication. Similarly the reduction of a fraction to lowest terms is illustrated.

Content of F. 26: This film illustrates the abstract concepts of division of fractions by the animation of familiar objects such as disks and doughnuts. The meaning of division by whole numbers is explained by relating it to measurement. The division of a whole number by a fraction is also demonstrated by actual measurement. From these problems, the rule for dividing by a fraction is developed step by step in a manner which explains why inverting the divisor and multiplying gives the correct answer easily. This rule is then applied to several problems one of which results in a fractional answer. Application of division by a fraction is made to the cutting of two yards of ribbon into bows ($2 \div \frac{2}{3}$) and to the making of a scale model airplane ($\frac{3}{4} \div \frac{3}{16}$).

Content of F. 27: This film uses familiar objects such as chicks and marbles that have pupil appeal to review the meaning of multiplication of whole numbers. A disk is used to show the multiplication of a whole number by a fraction. Segments of a disk are used to illustrate what happens when the numerator and denominators are multiplied together. Following the solving of a series of problems, the rule for multiplying fractions is introduced and demonstrated. The painting of a back-yard fence illustrates the application of multiplication by fractions.

Appraisal of F. 25: This film should improve the understanding of fractions since the principles are related to concrete objects. However, the film should not supplant the use of concrete materials by the teacher. It will be useful in setting an example of one way to use concrete material to make abstract concepts meaningful. This film seems to present the subject slowly enough so that it is suitable for initial teaching or remedial instruction. It could have included instruction on how to select the divisor for reduction. Not all mathematics teachers will agree that neatness is the major reason for reducing fractions to lowest terms.

Appraisal of F. 26 and F. 27: These films picture the explanations and concrete materials that good teachers should be using in the teaching of these concepts. Thus, these films will be very usable for teacher training purposes. But even if the teacher has made these concepts meaningful, the film should increase the pupils' general understanding of the principles of fractions, and make it easier for them to remember the rules of multiplication and division by fractions. It is commendable that these films do not present the process of cancellation. It is likely that very few pupils will be able to follow the explanation of the meaning of a fraction divided by a fraction. The films are designed for upper elementary grades, but could be used for remedial work or review in higher grades. They are not intended to be used to introduce multiplication or division of fractions. A teacher's guide is available for

each film. The photography is very good and the content is usually appropriate although some applications are not realistic. More variation from disks to actual life-like objects is needed to increase the interest if the entire series of films is to be used. Separately this is not monotonous, but collectively its effectiveness may be reduced. (F. 26 and F. 27 Reviewed by George McCuthcheon, Minneapolis.)

F. 28—*The Meaning of Percentage*

Young America Films, Inc.; 18 East 41st Street; New York 17, N. Y.
16 mm. sound film; 1 reel, 11 minutes; black and white—$38.50. Teacher's guide included. Technical Advisors: Brownell and Eads.

Description: The film opens with a scene of two children looking in shop windows where they see a number of sales signs using percentage. This is followed by a succession of scenes showing familiar social situations such as a school test paper, involving percentages. It relates the meaning of percentage to hundredths both as a fraction and as a decimal. Percentage as hundredths is shown graphically by a large square divided into 100 small squares. Percentage as a fraction is shown graphically by a parking lot that has a capacity of 100 cars. A comic sequence involving eight men is used to relate per cents to fractions such as one-fourth, one-half, three-fourths, and four fourths. The film ends with narration reviewing the symbols and meaning of percentage and its use in daily life.

Appraisal: This film should furnish a good introduction to use and meaning of percentage and its relation to fractions and decimals. It will be useful also for review and remedial work although the typical grade-placement of this film is grades 6–7. It should not supplant the use of objective material. The animated comic sequence in which a small pixie figure flies into the scene and reduces tall men to short, squat figures is weak. The narration and sequence of scenes has good continuity in the development of ideas, but may lack interest for elementary students

Technical Qualities: Photography: Very good. Sound: Commentary clear and appropriate; very good background music.

F. 29—*The Teen Numbers*

Young America Films, Inc.; 18 East 41st Street; New York 17, New York
16 mm. sound film; 1 reel, 10 minutes; black and white, $38.50. Teachers Guide included.
Technical Advisors: Brownell and Eads.

Description: The film opens on a scene in a small store where a young girl purchases a set of jacks with ten pennies. The counting of these ten pennies furnishes a background for developing the meaning of one-place numbers 1–9. As each of the numbers, 1 through 9, appears on the screen, small airplanes fly out and group themselves at the right of the number to remind the pupil of the meaning of each number. The meaning of the numbers 10–19 is developed as groups of ten and ones by showing ten airplanes parked on an airfield and other planes flying in to give the units. The numbers (ten and units) and the airplanes combine visually on the screen. Similarly a bundle of ten sticks and single sticks are combined to form numbers from 11 through 20. A column of numbers in serial succession is used to show how numbers are formed by using the digits 1–9 in different columns. Zero as a place holder is shown by combining ten with a one-place number to form a teen number.

Appraisal: This film may be used with groups of children who have learned to count to 20, who recognize the symbols through 20, and who have learned the meanings of the numbers through 10. The manipulation of objective material, such as marbles, should be combined with the showing of the film to help pupils recognize the serial and cardinal idea of numbers through 20. Although one of the combinations of digits by animation to form the teen numbers is weak in that it does

not show the first digit as a ten, this film should contribute much to making arithmetic interesting and meaningful. This film is appropriate for the second or third grade.

Technical Qualities: Photography: Very good animation. Sound: Clear, Appropriate Commentary. Excellent background music.

INSTRUMENTS

I. 6—*Globe-Hilsenrath Azimuth Computer*

Yoder Instruments; East Palestine, Ohio. Azimuth computer; $9\frac{1}{8}'' \times 9\frac{3}{4}''$; \$8.75 (educational discount)

Description: This computing instrument gives true azimuths if the latitude, declination, and hour angle are known. The following description is quoted from the booklet accompanying the instrument:

"The computer consists of a white Vinylite base and a transparent protractor of the same material. The base carries two printed grids—one on the front and one on the back. The settings for latitude and hour angle are made on these grids. In addition, there is a declination scale on each side. The azimuth is read from the protractor which is constrained, by a slot and stud, to slide from side to side along the base. There is an arrow on the protractor which extends over the declination scale. In use, the protractor is slid along the base until this arrow coincides with the declination desired. The front grid can be used between latitudes 45 North and 45 South. The back grid can be used between latitudes 40 and 60 North and between 40 and 60 South. The protractor can be transferred from front to back. A transparent arm is pivoted at the center of the protractor, which has a double row of figures. By a proper choice between the two rows, the value of the azimuth is obtained directly without conversion.

"A closer examination of the grids will reveal that they consist of a set of half-ellipses and half-hyperbolas. The semi-ellipses serves as latitude scales and the semi-hyperbolas are used as hour angle scales. In use, a point is chosen at the intersection of the ellipse for the latitude of the observer and the hyperbola for the hour angle of the body."

Appraisal: For anyone who has laboriously computed azimuths this will be a welcome and remarkably simple device. The only justification for introducing it in mathematics classes would be to illustrate mathematical formulas from spherical trigonometry and such advanced applications appear in only a few, specialized or advanced courses. However, since it is one of the most practical, ancient and important uses of trigonometry, it would seem that teachers might well attempt to use this device as enrichment material even though full understanding and complete skill in its manipulation are not objectives of the course. Even if it is not used in individual classes, it certainly would be an interesting and important addition to a school mathematics museum.

MODELS

M. 2—*John F. Schacht Dynamic Geometry Set*

W. M. Welch Mfg. Co.; 1515 Sedgewick Street; Chicago 10, Illinois. \$22.50 per set.

Description: The Dynamic Geometry Set consists of three devices which are described individually below:

"Triangle with constant midpoint" (Catalog number 7500). The instrument is essentially a triangular aluminum frame each of whose sides is painted a different color. By means of a clever brass gear attachment, each leg may be extended from a minimum length of $13\frac{1}{2}$ inches to a maximum of $22\frac{1}{2}$ inches. A 210-degree protractor is attached to each of the gears which remain at the midpoint of the triangle's sides; circular protractors are secured to each angle of the triangle. Accompanying each of the celluloid protractors is a tiny metal tab by means of which elastic bands may be extended from the midpoint of any

side to the opposite angle or to the other midpoints.

"Triangle with sliding point," (Catalog number 7505). This aluminum triangle has 16-inch legs—one red, another blue, the other yellow. 210-degree protractors are attached to metal runners which may be slid along each leg of the triangle; circular protractors are mounted in each angle of the instrument. Metal tabs attached to each protractor enable the user to extend elastics from one tab to any other.

"Quadrilateral with constant midpoint" (Catalog number 7510). The frame consists of a metal quadrilateral whose legs are red, blue, yellow, and unpainted aluminum. Circular protractors are mounted at each angle and at the midpoint of each leg which may be extended like a curtain rod from a minimum length of 14 inches to a maximum of 23 inches. Here again, metal tabs are fastened to each protractor.

The instruments are large enough to serve as good demonstration apparatus. They are excellently made and should, with proper handling, last indefinitely. Probably the most serious mechanical flaw is the poor visibility of graduations on the frames and protractors.

Educational Value: The set of instruments is extremely useful for the teaching of intuitive and demonstrative plane geometry. Properties of triangles and quadrilaterals may be demonstrated with the set but it may be used more advantageously if a sufficient number is purchased for use in a laboratory-type classroom. When the latter plan is adopted, students are in a position to profit from personal "discoveries" of geometric theorems and propositions. The manufacturers may have had this in mind when they formulated the price for the instruments.

The three instruments are appropriate for use on the secondary level. Students will enjoy manipulating them and recording their observations.

The prices of the aids are somewhat high but are not objectionable if the school budget is large for the devices are very good. (Reviewed by Bernard Singer, Boston, Massachusetts.)

TESTS

T. 1—*Lueck Algebra Readiness Test**

William R. Lueck; Public School Publishing Co.; 509–513 North East Avenue; Bloomington, Illinois.

Specimen set 46¢, per 25, \$1.80, key 18¢ extra; 1947; one form; 25 minutes; grade 8–9, just prior to the study of algebra; to help determine the probable success in algebra.

Description: Five short tests make up this test. The first contains 48 very simple arithmetic problems with integers in the fundamental operations. They are so easy that the student should be able to write the answers with very little effort. The second test has 15 problems which require the addition, subtraction, multiplication, and division of fractions and mixed numbers. The largest denominator is 16 and this appears but once. The third test, of 18 items, deals with similar operations with decimals. In each of these the numerical answer is given without the decimal point and some zeros. All that the student is required to do is to add any necessary zeros and the decimal point. The fourth test, of 20 multiple-choice items, is called

* The editors of this department of The Mathematics Teacher would be glad to hear comments on the desirability of including test reviews.

Device	Catalog Number	Quant. 1–2 Price	Quant. 3–9 Price	Quant. 10–24 Price	Quant. 25–99 Price	Quant. 100 Price
Triangle with constant midpoint	7500	7.50	6.00	5.50	5.00	4.50
Triangle with sliding point	7505	6.50	5.00	4.50	4.00	3.50
Quadrilateral with constant midpoint	7510	8.50	7.00	6.50	6.00	5.50

problem-solving. The verbal problems are short arithmetic problems of the type usually found in the 7th and 8th grades. Four possible responses are given. The last test, called General Numbers, presents an easy lesson on the use of letters in performing the four fundamental operations. There follows 18 questions and problems based on this lesson. The test is printed on a large 4-page folder. Test 1, on the front page, is reversed so that the paper must be turned up-side-down after filling in the usual blanks at the top. Whether or not this prevents all of the students from looking at the problems in advance is questionable. They could be easily read if up-side-down and such an arrangement might serve as a challenge to some pupils. The printing and other arrangements are good. In Test 2, Problem 8, there is an extra point which might cause confusion. The problem reads $2\frac{3}{4}\times 8$, but there is a dot before the 8, although somewhat lower than normal for a decimal point.

Validity. In one school, the coefficient of correlation between scores on this test and teachers' grades was .78 for 112 cases. In three schools with 126 cases, a similar coefficient of .64 was found. This test is the result of several years' work and experience in attempting to predict success in algebra. Most of the content is arithmetical, which would seem to indicate that if this test is successful in its objective, a good achievement test in arithmetic also might be successful.

Reliability. A coefficient of reliability of .96 was found by the odd-even method and the Spearman-Brown formula. The size of the sample was not given.

Administration. The five tests have definite time limits of 2, 4, 4, 8 and 8 minutes, respectively. The directions require that scratch paper be passed out with the booklets and be placed over Test 1 on the front page, so that only the blanks to be filled in are showing. This would seem to be an invitation to "peek." Also it makes distribution more complicated. Verbatim instructions are printed in the manual. The scratch paper is the only extra material needed.

Scoring. A key is provided on a sheet of very heavy paper. Test 1 is scored by means of a stencil arrangement. The key is to be placed over the test so that the answers are immediately over the student's answers. The keys for the other four tests have been arranged along the four sides of the sheet. These line up with the spaces provided for the answers. The score on Test 1 is half the number right while, on Test 5, it is twice the number right. The scoring is sufficiently objective so that anyone should be able to correct the test. A class record sheet is available.

Interpretation. Scores on each of the five tests and the total which corresponds to each 5 per cent are given. These have been based on data from 578 cases. These norms are for pupils taking the test in September, but they should not vary greatly from the scores obtained from giving it in May or June. The manual states that students scoring above the 40th percentile usually pass in algebra, those between the 25th and the 40th percentiles usually have difficulty and are border-line cases for whom other factors should be carefully considered. Those below the 25th percentile usually fail to profit from algebra, and other factors should be most favorable before these are allowed to study the subject. No proof of the accuracy of these levels is given, nor are there any figures on the numbers below the critical score who pass, or the number above it who fail and are so reported.

Conclusion. This is a very short test on which to base any important decisions. It must be used only in conjunction with all other factors. The predominance of the usual arithmetic work makes one wonder why it is superior for its purpose to any good achievement test in arithmetic. (Reviewed by John H. Haynes, Newton, Massachusetts.)

◆ IN OTHER PERIODICALS ◆

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